Restriction of Qualitative Models to ensure more Specific Behavior

David Hibler
Dept. of Computer Science
Christopher Newport College
Newport News, VA 23606.
tele: (804)-594-7360
e-mail: dhibler@pcs.cnc.edu

Gautam Biswas
Dept. of Computer Science
Vanderbilt University
Nashville, Tennessee 37235.
tele: (615)-343-6204
e-mail: biswas@vuse.vanderbilt.edu

April 16, 1991

Abstract

This paper discusses the restriction of qualitative models of physical systems in order to ensure more specific behavior. Restriction is performed by adding constraints: (i) directly on exogenous parameters, and (ii) on non exogenous or dependent parameters. The former are termed *primary* restrictions, and the latter are called *derived* restrictions. We show the connection between primary and derived restrictions, and how to determine the consistency of multiple restrictions. Examples are given to show how a specific model is augmented using restrictions.

1 Introduction

A serious problem with current methodology for qualitative behavior generation is ambiguity [11]. Qualitative models may produce multiple behaviors due to lack of precision in the model. A model of a refrigerator should generate behavior that implies heat transfers from the cold side (i.e., inside of the refrigerator) to the hot side (i.e., the outside). A model of a heat exchanger should derive heat transfer from the hot side to the cold side. A qualitative model intended to represent a refrigerator should not allow heat exchanger behavior to dominate and vice versa. Very often multiple behaviors that are generated are due to:

1. the fact that a parameter value depends on the difference in two other parameter values, but this cannot be unambiguously determined from their current qualitative values, and

2. the relative rates at which interacting parameter values change is not known, therefore, if behavior descriptions depend on relative rates this will invariably cause branching behavior.

Note that both these problems would not exist if more precise quantitative models were employed. In qualitative simulation the second problem has been addressed by ignoring irrelevant distinctions[10], and by incorporating higher-order derivatives of system parameters[3,10,12]. The first problem, which is purely an artifact of the qualitative nature of parameter descriptions, and not the structure of the model cannot be handled by the above mechanisms in an elegant way. For example, if such ambiguities occur when analyzing the behavior of a device whose properties are not well known to the user, there is no way to read the modeler's (i.e., the designers) mind to determine which behavior is "correct" and which behavior is an artifact produced by the qualitative nature of parameter definitions.

Our answer to the ambiguity problem is to refine the qualitative structure of the model to generate the desired behavior. The methodology developed examines the qualitative constraints which have to be added to the model in order to *restrict* the behavior in the desired way. These additional constraints are explicitly justified by propagating them back in a specific way to constraints on *exogenous* parameters. Exogenous parameters are "givens", i.e., they are causally external to the model. The propagation is achieved by principles based on *continuity* and *monotonicity* properties. Linking behavior to structure is achieved by explicitly "justifying" the initial set of constraints in terms of constraints on exogenous parameters. As a final step, we add the constraints on the exogenous parameters to augment the original model. In general, we may impose any consistent set of constraints on the exogenous parameters, and usually there are different exogenous parameters that may be constrained in different ways to produce a more specific qualitative model. However, certain restrictions may be more realistic or useful than others.

Let us consider a simple example involving a container holding a gas at a pressure P. If $P \geq P_o$ the container breaks, and if $P < P_o$ the container does not break. $P = f_1(T)$, where T is the temperature, and $P_o = f_2(S)$, where S is the strength of the container. T and S are the independent (exogenous) variables.

We wish to restrict the model to one which does not break. To guarantee the proper behavior, we can simply add $P < P_o$ as an assumption to the model. However, this re-

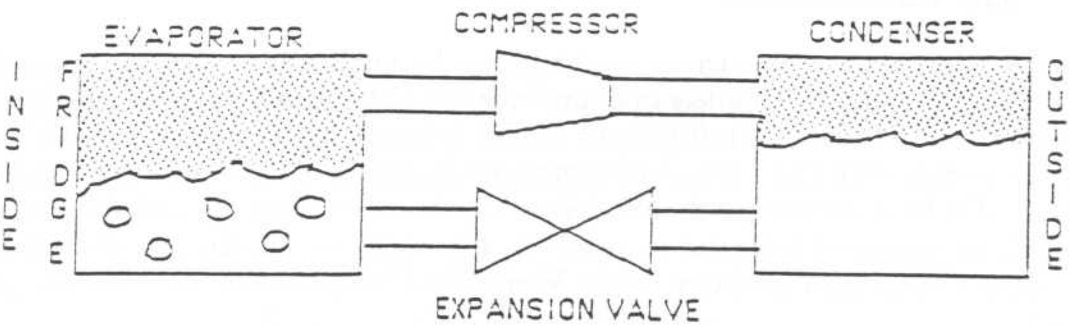

Figure 1: Schematics of the Refrigerator

quirement is not useful for either explanation or control purposes. Both P and P_o are determined by other parameters, therefore, they cannot be considered as primary causal factors for explanatory purposes. In our example system, they are also not directly controllable. Therefore, the constraint $P < P_o$ needs to be propagated back to the exogenous parameters. In this case, it is equivalent to requiring that T and S satisfy $f_1(T) < f_2(S)$. To make this constraint operational, it is then necessary to derive an interval (in general, a connected region in the space of exogenous variables) where the above inequality holds.

Another issue relates to which exogenous parameter the constraint should be related to. For one, we consider only those exogenous parameters that can affect variables in the original constraint. may be that the original constraint may be propagated back to one parameter but not to the others. If we know that f_1 is monotonically increasing from zero we can express the constraint $f_1(T) < f_2(S)$ as $T < f_3(S)$ where f_3 is some function of S. On the other hand, if we know nothing about f_2 we cannot express the constraint in terms of S directly. Another reason for expressing a constraint in terms of particular exogenous parameters involves realistic choices concerning which exogenous parameters are really at our disposal. Suppose that we know that f_2 is also monotonically increasing from zero. We can express $f_1(T) < f_2(S)$ either as $T < f_3(S)$ or $S > f_4(T)$. One of these restrictions will usually be more realistic than the other, however. For example, from a practical standpoint the temperature may be the exogenous parameter that is easier to control, therefore, $T < f_3(S)$ is the explicit constraint that gets added to the model definition.

In the remainder of this paper we develop the theoretical principles of restriction and illustrate them in connection with the refrigerator problem. In the last section we compare our work with other approaches, primarily the OPERATING ASSUMPTIONS of Falkenhainer and Forbus[4], and discuss future work.

2 The Refrigerator Example

To illustrate the applicability of the theory developed, the restriction process has been applied to a qualitative model of a refrigerator (Fig. 1) to generate more specific behavior[1, 7]. The model with restrictions is implemented in Prolog on a problem solver called TEPS (*Thought Experiment Problem Solver*).

The refrigerator model shown in Fig. 1 has six primary locations:

Evaporator - ev	Compressor - cm
Condenser - cn	Expansion Valve - ex
Interior of Refrigerator - int	Exterior of Refrigerator - ext

The interior and exterior of the refrigerator are modeled as heat sources or sinks. Collins and Forbus[1] and Hibler and Biswas[7] have demonstrated that by assuming multiple ontologies one can establish that under certain conditions the refrigeration system in equilibrium pumps heat from a cooler to a higher temperature, i.e., the system actually cools. This requires the use of general processes[1], such as *boiling, condensing, heat flow, and fluid flow*, and more specialized processes such as *throttling* that takes place between the expansion valve and the evaporator. In the rest of this paper we refer to different aspects of the refrigeration system to illustrate various concepts that are associated with restriction principles.

3 Basic Definitions

As discussed earlier, the key step in restricting system behavior is to impose additional constraints on system parameter values. If these parameter values are "causally external" (i.e., exogenous) to the system and create no inconsistencies the problem is solved. However, if the restricted parameter is a *dependent* parameter, it needs to be linked to exogenous parameters through a dependency graph.

3.1 Exogenous and Dependent Parameters

Intuitively, a parameter is exogenous if its value is independent, i.e., it does not depend on values of other parameters that make up the system. If we take the point of view of someone designing a physical system we call exogenous parameters *design parameters*, and we call the added constraints *design choices*. The constraints are sufficient (but perhaps not necessary) to ensure a particular behavior (design behavior).

More formally, in the QPT framework we define two types of exogenous parameters. The first type of exogenous parameter is a *variable* which is exogenous because it has *no influences* on it in the given scenario model. In the refrigerator example, the exterior and interior of the refrigerator are both modeled as heat sinks with some given fixed temperature. Thus T_{int} and T_{ext} are exogenous parameters of the first type.

The second type of exogenous parameter is an *initial condition* on a directly influenced variable. In QPT a variable may be directly or indirectly influenced, but not both. Direct influences on a variable correspond to a qualitative version of a first order differential equation[5]. The initial choices for the variable are arbitrary, and can be set externally from the scenario model. In the refrigerator model the evaporator temperature, T_{ev}, is influenced by heat flow. Its initial value, however, can be set arbitrarily. On the other hand, indirect influences correspond to functional relationships so initial choices of indirectly influenced variables are not arbitrary. The saturation temperature, Ts_{ev}, of the evaporator is functionally determined by evaporator pressure, P_{ev}.

A variable that is not directly influenced may have functional relationships (i.e., qualitative proportionalities) and correspondences to other variables[5]. For purposes of analysis,

[1]We are working in the QPT framework[5]

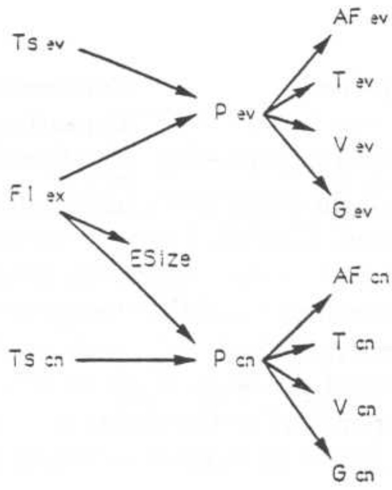

Figure 2: Partial Dependency Graph for the Refrigerator

such parameters need to be functionally related to exogenous parameters.

3.2 Relating Dependent Parameters to Exogenous Parameters

Given a set of qualitative proportionalities: α_q, α_{q+}, α_{q-} which hold between the quantities of the model we can construct a *dependency graph* with the quantities as nodes and the qualitative proportionalities between quantities specifying directed edges. The direction of the edge is in the direction of dependence, i.e., if $A\ \alpha_q\ X$, then $A \rightarrow X$. [2] QPT model formulations allow no circular dependencies, therefore, the dependency graph is a directed acyclic graph (see Fig. 2). Since there are a finite number of parameters the graph is finite. Given any variable which is functionally dependent on other quantities we follow all paths from that variable until we reach leaves of the graph. These leaves represent variables which ultimately determine the value of the given variable. Since they are leaves they are not functionally dependent on any parameter. This means they must be exogenous parameters or directly influenced parameters[3]. In either case, we can choose their initial values arbitrarily. If they are exogenous parameters we can even choose their values as arbitrary functions of time. Since we are restricting an already existing model the dependency graph is easily found by examining the collection of all qualitative proportionalities which hold for a given process and view structure of the scenario model. A part of the dependency graph for the refrigerator problem appears in Fig. 2.

Given a functionally dependent variable we can collapse its dependency graph by substituting functions for intermediate variables in the path to the leaves. The result is a composite function whose arguments are the exogenous variables. This composite function can be described by qualitative proportionalities. We refer to these as the *ultimate* qualitative proportionalities for a variable. Derivation of ultimate qualitative proportionalities from the component qualitative proportionalities are discussed in Section 4.

[2]The direction of the arrow may be the reverse of what is chosen by convention. We chose this convention initially, and have stuck to it.

[3]Note that rates of directly influenced parameters are also functions of other parameters; this issue is discussed later. For now, we consider only qualitative proportionalities

3.3 Restrictions

Constraints which must be added to a model to ensure a desired behavior are termed *restrictions*. We are interested, not only in the constraint being satisfied but in "how" it can be satisfied. We view this as solving the constraint in a particular direction. Thus, one or more of the variables are now dependent on the others. This restriction is expressed by an equality or inequality involving the restricted variable. For example, if the constraint involves X, Y, and Z we might restrict X in order to satisfy the constraint so that $X > F_1(Y, Z)$. This is needed because we want to impose specific dependency relationships on an exogenous parameter.

Restrictions directly on exogenous parameters are termed *primary* restrictions. In general, the parameters involved in forming restrictions need not be exogenous. Restrictions on non exogenous variables are called *derived* restrictions. Derived restrictions often arise because a certain component within a device has to be forced to function in a certain way (e.g., a transistor in an electronic circuit has to remain in saturation) to achieve a desired overall behavior (or to ensure that certain behaviors do not occur). Derived restrictions have to be justified by linking them to primary restrictions by a formal methodology. This requires determining exogenous parameters from a scenario model, establishing primary and derived restrictions and their relationships, and checking if a set of restrictions added to achieve a specific behavior are consistent.

In the refrigerator example there are a number of restrictions which must be made in order to obtain an equilibrium state in which the model is actually functioning as a refrigerator. First, it is necessary to ensure that the initial conditions are such that heat flow occurs in the proper direction, i.e., from the inside to the outside of the refrigerator. Correct functioning of the the condenser and evaporator are the key to ensure this behavior. Second, it is necessary to ensure that the refrigerator continues to operate properly. Thus we must have equilibrium conditions such as balanced fluid flow and a constant gas fraction in each location.

To elaborate, a trivial restriction that we can establish is that the refrigerator operates by taking heat from a cold interior to a warmer exterior, i.e., $T_{int} < T_{ext}$. This restriction is primary since both parameters are exogenous in our model. To ensure heat flow into the evaporator from the interior of the refrigerator we must choose $T_{ev} < T_{int}$. This is still possible since T_{ev} is exogenous.

An example of a derived restriction is the requirement that the temperature of the evaporator be at the saturation temperature (boiling point): $T_{ev} = Ts_{ev}$ so as to ensure that the fluid absorbs heat as it evaporates and expands. The temperature of the evaporator is already constrained by $T_{ev} < T_{int}$. Ts_{ev}, on the other hand, is not exogenous. It is determined by the pressure in the evaporator which in turn depends on other quantities (see Fig. 2). To constrain Ts_{ev} by the above equality, we must relate it to restrictions on exogenous quantities.

3.4 Ways of Implementing Restriction

The restriction method constitutes a theoretical technique which aids in constructing qualitative models. The implementation of this method in a problem solver can be at various levels of detail. At the simplest level the necessary restrictions as well as their justifications are simply built into the model as *assumptions*. This method will be termed *basic* *restriction.* This method has the advantage that it can be added to a qualitative model with little change in the way the problem solver operates. Qualitative simulation using the model verifies that the restrictions produce the desired behavior. A second method is *verified restriction.* Here the necessary restrictions, both primary and derived are added to the model; however, the justifications of one in terms of the other are verified by the problem solver to prevent the possibility of human error in constructing the model. Yet a third method is *automatic restriction.* In this case restrictions would be determined automatically from specifications of desired behavior.

4 Composition Rules for α_q's

Composition of a sequence of α_q functions can be summarized into the following rules:

1. The composition of any number of α_{q+}'s produces a resultant α_{q+}.
2. The composition of an odd number of α_{q-}'s produces a α_{q-}.
3. The composition of an even number of α_{q-}'s results in a α_{q+}.
4. Any composition involving a α_q results in a α_q.

The conditions under which the *ultimate* α_q value holds is determined by the logical "and" of all the conditions under which each of the individual α_q values were deemed true.

An example of composition used in the refrigerator model involves the saturation temperature Ts_{ev}. This depends only on the pressure, i.e., $Ts_{ev}\ \alpha_{q+}\ P_{ev}$ (Fig. 2). In our model, the pressure in a container depends on temperature, T, amount of fluid, AF, fraction of fluid which is gas, G, and volume of the container, V.

$$P_{ev}\ \alpha_{q+}\ AF_{ev},\ P_{ev}\ \alpha_{q+}\ T_{ev},\ P_{ev}\ \alpha_{q-}\ V_{ev},\ P_{ev}\ \alpha_{q+}\ G_{ev}$$

By composition with the previous proportionality we obtain the ultimate qualitative proportionalities:

Relation	Type of right hand side	Influences
$Ts_{ev}\ \alpha_{q+}\ AF_{ev}$	(initial exogenous)	Fluid flows
$Ts_{ev}\ \alpha_{q+}\ T_{ev}$	(initial exogenous)	Heating
$Ts_{ev}\ \alpha_{q-}\ V_{ev}$	(exogenous)	none
$Ts_{ev}\ \alpha_{q+}\ G_{ev}$	(initial exogenous)	various

The issue that remains to be resolved is what happens when there are multiple paths from a dependent variable node to a given leaf. Forbus considers the dependency graph to always be a tree so this case would not occur[5]. We prefer to leave this possibility open. If there are multiple paths and the ultimate α_q value of each path is the same type, then the ultimate α_q value between dependent variable and leaf node variable is of this type. If there are multiple paths and the α_q values along each path are different, then the ultimate α_q value between the two variables is *indeterminate.*

5 Relating Primary and Derived Restrictions

Derivation of parameter restrictions for dependent variables require that they be linked to restrictions on related exogenous variables (i.e., primary restrictions). This may require that the quantity spaces of the exogenous variables involved be augmented by new landmark values.

Basically we would like to use properties of monotonic increasing or decreasing functions to relate restrictions of dependent variables to restrictions on independent variables. In QPT however we have qualitative proportionalities instead of directly having monotonic functions. A qualitative proportionality expresses a functional relationship, and provides a constraint on the partial derivative of any such function, i.e.,

$$Y \ \alpha_{q+} \ X \ \Rightarrow \ Y \ = \ f(\cdots, X, \cdots), and \frac{\partial f}{\partial X} \ > \ 0.$$

It does not specify the complete list of arguments in the functional relationship.

We also are faced with the need to work backward. We have restrictions on dependent parameters which we would like to translate into restrictions on exogenous parameters. Since the original parameters are dependent we have ultimate qualitative proportionalities which relate them to exogenous parameters. The range for the dependent variable for which a particular qualitative proportionality holds is important. We are trying to restrict the dependent variable to a certain value or range of values. This means that the qualitative proportionalities which are of interest are those which hold in this restriction range. For example, assume we are trying to justify the derived restriction $A > B$ where A is the dependent variable. The restriction range for A is the open interval (B, ∞). Knowing that $A \ \alpha_{q+} \ X$ in some region below B is irrelevant.

For consistency of notation we add to the quantity space for A, the quantities $+\infty$ and $-\infty$. We add the order relations $X < +\infty$ and $X > -\infty$ for every value in the old quantity space including A. We also adopt the () notation for an open interval in a quantity space, i.e., $A \in (A_1, A_2)$ means $A > A_1$ and $A < A_2$.

It is useful to go back and examine the characteristics of the exact models which we can represent using a qualitative proportionality. In each of those models A is given by a continuous, strictly increasing function, $f(\cdots, X, \cdots)$ in some domain, D which is a subset of the real line. The range of f is (A_1, A_2). If such an f did not exist there would be some point A_3 in (A_1, A_2) for which the qualitative description $A \ \alpha_{q+} \ X$ (or $A \ \alpha_{q-} \ X$) would fail. Although this fact may seem obvious it is not totally trivial. There can actually be several such functions corresponding to different domains. For example, if $A = sin(X)$ and we are concerned with A in the interval $(0,1)$ there are actually an infinite number of X intervals for which $A \ \alpha_{q+} \ X$. We have a different, continuous strictly increasing function from each of these intervals to $(0,1)$.

It is an elementary theorem that the inverse of a continuous strictly increasing function is a continuous, strictly increasing function. Thus f has an inverse $finv$. Furthermore, the image of (A_1, A_2) under this inverse is an open interval (X_1, X_2). The domain of our qualitative proportionality is described by landmarks and certainly does not include a landmark for every point in (X_1, X_2); we can, however, add further landmark values to the quantity space for X which correspond to any of these points. The only requirement on a new landmark value is that such a value must exist in every exact model which is represented by our qualitative model.

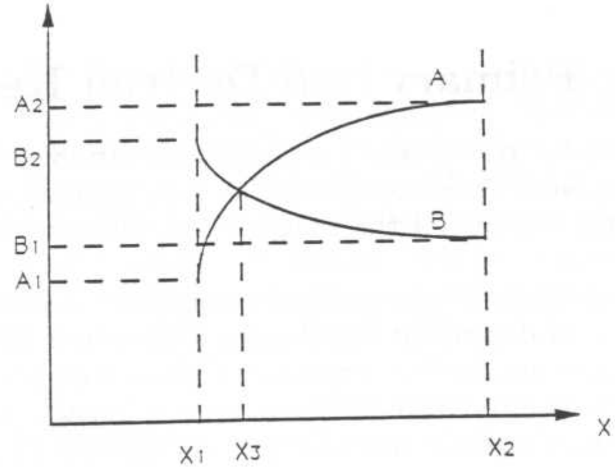

Figure 3: Behavior Of A And B With Respect To X

We would like to use continuity and asymptotic behavior to propagate derived constraints of the form $A > B$ or $A < B$ or $A = B$ back to constraints on exogenous parameters. For example, in the refrigerator model, total flow of fluid is determined by competition between compressor and expansion valve. We choose models where the compressor runs at a constant rate, but the expansion valve opens and closes depending on the pressure in the condenser and evaporator. From a control viewpoint, the expansion valve must determine the fluid flow rate when it is open, and the derived restriction $Fl_{ex} > Fl_{cm}$ has to be imposed so that the amount of fluid in the evaporator does not decrease to zero. Note that Fl_{ex} and Fl_{cm} are flow rates in the expansion valve and compressor, respectively. Fl_{ex} depends on several quantities, one of which is a size parameter for the expansion valve opening: $Fl_{ex}\ \alpha_{q+}\ ESize$. By propagating the derived constraint back to a constraint on this size parameter we can ensure that it is large enough so that $Fl_{ex} > Fl_{cm}$.

The basic idea when using α_{q+} to justify a derived constraint is that if B is in the open interval (A_1, A_2) and is not increasing with X while A does increase with X we must have the situation shown in the graph in Fig. 3. This means we can ensure $A > B$, $A < B$ or $A = B$ with the proper choice of X. A similar situation holds when using α_{q-}.

The following lemmas can be rigorously established.

Restriction Derivation Lemmas

Lemma 1. $A > B$ justification by $X > X_3$ when $A\ \alpha_{q+}\ X$
Assume:
a. $A\ \alpha_{q+}\ X\ forA\ \in (A_1, A_2)$
b. $B\ \alpha_{q+}\ X$ is not true if $B \in (A_1, A_2)$
c. $B < A_2$, and d. X is exogenous.
Then:
We can add an element, X_3, to the quantity space for X such that a choice of the primary restriction, $X > X_3$, implies the derived constraint, $A > B$.

Lemma 2. $A < B$ justification by $X < X_3$ when $A\ \alpha_{q+}\ X$
Assume: a. $A\ \alpha_{q+}\ XforA \in (A_1, A_2)$
b. $B\ \alpha_{q+}\ X$ is not true if $B \in (A_1, A_2)$
c. $B > A_1$

d. X is exogenous.
Then:
We can add an element, X_3, to the quantity space for X such that a choice of the primary restriction, $X < X_3$, implies the derived constraint, $A < B$.

Lemma 3. $A > B$ justification by $X < X_3$ when $A\ \alpha_{q-}\ X$
Assume:
a. $A\ \alpha_{q-}\ X for A \in (A_1, A_2)$
b. $B\ \alpha_{q-}\ X$ is not true if $B \in (A_1, A_2)$
c. $B < A_2$
d. X is exogenous
Then:
We can add an element, X_3, to the quantity space for X such that a choice of the primary restriction, $X < X_3$, implies the derived constraint, $A > B$.

Lemma 4. $A < B$ justification by $X > X_3$ when $A\ \alpha_{q-}\ X$
Assume:
a. $A\ \alpha_{q-}\ X for A \in (A_1, A_2)$
b. $B\ \alpha_{q-}\ X$ is not true if $B \in (A_1, A_2)$
c. $B > A_1$
d. X is exogenous.
Then:
We can add an element, X_3, to the quantity space for X such that a choice of the primary restriction, $X > X_3$, implies the derived constraint, $A < B$.
Lemma 5. $A = B$ justification by $X = X_3$ when $A\ \alpha_{q+}\ X$.
Assume:
a. $A\ \alpha_{q+}\ X for A \in (A_1, A_2)$
b. $B\ \alpha_{q+}\ X$ is not true c. $B \in (A_1, A_2)$
d. X is exogenous.
Then:
We can add an element, X_3, to the quantity space for X such that a choice of the primary restriction, $X = X_3$, implies the derived constraint, $A = B$.
Lemma 6. $A = B$ justification by $X = X_3$ when $A\ \alpha_{q-}\ X$
Assume:
a. $A\ \alpha_{q-}\ X for A \in (A_1, A_2)$
b. $B\ \alpha_{q-}\ X$ is not true c. $B \in (A_1, A_2)$
d. X is exogenous.
Then:
We can add an element, X_3, to the quantity space for X such that a choice of the primary restriction, $X = X_3$, implies the derived constraint, $A = B$.

The proofs of these Lemmas appear elsewhere.

In the refrigerator model we wish to justify the derived restriction $Ts_{ev} = T_{ev}$ where $Ts_{ev}\ \alpha_{q+}\ AF_{ev}$. The interval for Ts_{ev} is $(0, \infty)$. AF_{ev} plays the role of X. We use lemma 5 and require $AF_{ev} = AF_1$ for justification.

Lemmas 1–6 did not specify how X_3 depends on the other quantities. In each case we held fixed all dependencies except those on X. The other quantities involved are A_1, A_2, A, and B. Let us consider A_1 and A_2 to be fixed. We, therefore, take the set of all parameters on which A, and B depend and delete X. The resulting set is the set of parameters on which X_3 will depend. In the above example the primary restriction is actually $AF_{ev} = AF_1(T_{ev}, V_{ev}, G_{ev})$. We can actually determine more details of the parameter dependencies in primary restrictions. The primary restriction in any corresponding exact model can be written as an equality involving a new parameter K_1. $X = X_3 + K_1$, where K_1 is required to be $+, -$, or 0 as needed to provide the $>, <$, or $=$ when K_1 is not present. Likewise the derived constraint may be written $A = B + K_2$, where K_2 is $+, -$, or 0 as required. Thus $dA = dB + dK_2$

If $A = A(X, S_1, \cdots, S_n)$ and $B = B(X, T_1, \cdots, T_m)$ then the above equation which becomes

$$(\frac{\partial A}{\partial X})dX + (\frac{\partial A}{\partial S_1})dS_1 + ... + (\frac{\partial A}{\partial Sn})dS_n = (\frac{\partial B}{\partial X})dX + (\frac{\partial B}{\partial T_1})dT_1 + ... + (\frac{\partial B}{\partial T_m})dT_m + dK_2,$$

where $\frac{\partial F}{\partial G}$ means the partial derivative of F w.r.t. G. This leads to

$$dX3 = \frac{a}{(\frac{\partial B}{\partial T_1})dT_1 + ... + (\frac{\partial B}{\partial T_m})dT_m - (\frac{\partial A}{\partial S_1})dS_1 - ... - (\frac{\partial T}{\partial S_n})dS_n} \; - \; dK_1 \; + \; a \cdot dK_2,$$

where $a = \frac{1}{((\partial A/\partial X)-(\partial B/\partial X))}$.

By examination of the above equations we can determine four rules for qualitative proportionalities involving X_3. These dependency rules are:

Restriction Dependency:

1. If either A or B in the lemmas above depends on a parameter in an unknown way, then X_3 depends on the same parameter in an unknown way.
2. For justification lemmas using α_{q+}:
   If B depends on a parameter, X_3 depends on that parameter in the same way.
   If A depends on a parameter, X_3 depends on that parameter but in the opposite way.
3. For justification lemmas using α_{q-}:
   If A depends on a parameter, X_3 depends on that parameter in the same way.
   If B depends on a parameter, X_3 depends on that parameter in the opposite way.
4. In case of conflict using the above rules a dependency exists but is of unknown type.

In the refrigerator example, Ts_{ev} is A, T_{ev} is B, and AF_{ev} is $X3$. Since $Ts_{ev}\ \alpha_{q-}\ V_{ev}$ we must have $AF_{ev}\ \alpha_{q+}\ V_{ev}$.

5.1 Consistency of Multiple Restrictions

We can combine primary restrictions if they are consistent. This, of course, means the corresponding derived restrictions must hold and be consistent.

Primary restrictions are of one of the following three types:

(I) $X > X_1(R_1, \cdots, R_n)$
(II) $Y < Y_1(S_1, \cdots, S_m)$

(III) $Z = Z_1(T_1, \cdots, T_w)$

The left sides represent the dependent quantities; the right sides contain the independent quantities in the given restriction. To determine the consistency of the set of restrictions do the following –

First for each restricted parameter determine whether the set of restrictions for it is consistent using the following rules:

1. Any two nonidentical restrictions of type three are inconsistent.
2. Any set of restrictions on the same parameter which are all of type one or all of type two are consistent.
3. Two restrictions of different types are of unknown consistency unless there is special information in the form of an auxiliary inequality relating the right hand sides of the restrictions, i.e.,
   if $X > X_1$ and $X < X_2$ then we need $X_1 < X_2$
   if $X > X_1$ and $X = X_2$ then we need $X_2 < X_1$
   if $X < X_1$ and $X = X_2$ then we need $X_2 < X_1$
   The needed auxiliary inequalities may be known. If not, they must be taken as additional derived constraints and we must attempt to find choices for primary restrictions which justify them. In special cases this works but not in general.

If each pair of restrictions on a parameter are consistent then the set of restrictions on that parameter is consistent. If any pair of restrictions on a parameter is inconsistent then the set of restrictions is inconsistent. Otherwise, consistency of the set of restrictions on a single parameter is unknown.

To show that a given set of restrictions on multiple parameters is consistent we determine if the restrictions on each parameter are consistent. If they are, then we construct the dependency graph for all restrictions on all parameters. A sufficient condition for consistency of the entire set of restrictions is that this graph have no cycles.

If the dependency graph for restrictions has cycles, they can often be broken by compensation. For example, if $A\ \alpha_{q+}\ X$ and $A\ \alpha_{q+}\ Y$, and these proportionalities have the same range, we can force A not to be dependent on X by adding the derived constraint $A = B$, where B is constant. This is nothing more than an application of Lemma 5. It is called compensation since the primary restriction which ensures $A = B$ will force Y to depend on X in such a way that it compensates for changes in X.

5.2 Restrictions and Time Behavior

The correct time behavior of a system may occur automatically if the correct restrictions are made at an initial instant of time. We can also restrict the time behavior more directly by restricting the values which can be assumed by the magnitude of the derivatives of quantities of interest. The magnitude of a derivative can be equated to a separate variable. The methods discussed above will apply to this variable and we can restrict its values by the same methods we would with any other variable. If we define $Y = \frac{dX}{dt}$ then Y is just another parameter which we can restrict.

In certain cases, when the exogenous parameter is not of an initial value type the parameter value may be defined to vary as a function of time so that the restriction holds permanently. Using an exogenous parameter whose value varies as a function of time to ensure a desired behavior usually means that the system is actually coupled to some feedback device which is not modeled.

6 Conclusions

We have developed a technique for restricting qualitative models to ensure more specific behavior. The basic restriction technique was applied to a refrigerator model. It required fourteen primary and six derived restrictions in order to ensure that the basic model (a) transferred heat from cold to hot sides only and (b) started from and stayed in an equilibrium state[8].

The implementation method involved adding primary restrictions as assumptions to the individual view for the refrigerator. The individual view for the refrigerator also contained Prolog rules which produced the associated derived restrictions when the appropriate qualitative proportionalities were active and recorded the connection for possible explanatory or diagnostic use.

One solution to the ambiguity problem is to add to the model assumptions which select one of the possible behaviors. These additional assumptions are termed OPERATING assumptions by Falkenhainer and Forbus[4]. OPERATING assumptions are always consistent with the model in that they select one of the possible behaviors and do not impose a behavior which contradicts the model.

What is wrong with OPERATING assumptions? The difficulty is that they are *ad hoc* selection rules which are not explicitly connected to the structure of the model. Derivation of behavior from structure is more than a philosophical requirement; it makes the model more useful. For example, if our refrigerator fails to operate properly the restrictions should provide an aid in diagnosing the cause of failure. The restriction process can be an aid in developing qualitative designs for mechanisms and qualitative control methods. The specificity of restriction allows for more realistic models. Do we prevent a gas container from exploding by limiting the amount of gas it contains or by increasing the strength of the container as needed. Both parameters may be technically exogenous but limiting the amount of gas may be far more realistic than increasing the strength of the container.

A problem solver using basic restriction must explicitly contain the connections between primary and derived restrictions. This makes them available for diagnostic and explanatory purposes. It can, however, rely on the person constructing the model for the validity of the connections. Qualitative simulation with such a problem solver verifies that the restrictions produce the desired behavior.

A problem solver capable of restriction verification must be able to verify that the connections are valid ones according to restriction theory. It should perform the following steps:

1. Identify all exogenous parameters in the model it is using.
2. Verify the association of derived restrictions with primary restrictions using the rules 1-6 given.
3. Verify consistency of multiple restrictions.

Steps 1-3 are not difficult and has been added to the TEPS problem solver. Verification can be turned on or off; when it is on the problem solver examines all α_q's which are active for the current process and view structure, and constructs a dependency graph based on these. Leaves of this graph determine exogenous variables. The composition rules given earlier allow verification of ultimate qualitative proportionalities. Restrictions are labeled by type; consistency of restrictions of each variable is checked separately. Finally, a dependency graph for restrictions is created and checked for cycles.

Automatic restriction is a difficult problem. Often there are many possible ways of restricting a system with no unique choice. Furthermore, finding a set of restrictions which ensures a specified behavior by performing a systematic search through the space of all possible combinations of all possible restrictions seems hopelessly difficult in most cases. One possible approach involves the thought experiment method discussed elsewhere[7]. As applied to this problem the method would consist of using simplified versions of a problem, determining the restrictions needed to ensure a certain behavior for these simplified models then assuming that the restrictions required for the more complex model were a superset of those found for the elementary model.

The present paper has dealt with the restriction problem for qualitative models based on the QPT formalism. Qualitative Process Theory has the advantage that the basic formulation of a model uses functional dependencies are always "one way", leading to clear identification of exogenous parameters. If suitable information about exogenous parameters is added to models based on the techniques such as those of de Kleer and Brown[2], Kuipers[9] and others, similar results are possible.

References

[1] J. Collins and K.D. Forbus. "Reasoning about Fluid via Molecular Collections", *Proc. AAAI-87*, Morgan Kaufmann, San Mateo, CA, pp. 590–595, 1987.

[2] J. de Kleer and J. S. Brown. "A qualitative physics based on confluences", *Artificial Intelligence*, vol. 24, pp. 7-83, 1984.

[3] J. de Kleer and D.G. Bobrow. "Qualitative Reasoning with Higher-Order Derivatives", *Proc. AAAI-84*, Morgan Kaufmann, San Mateo, CA, pp. 86-91, 1984.

[4] B. Falkenhainer and K.D. Forbus. "Setting up Large-Scale Qualitative Models", *Proc. AAAI-88*, Morgan Kaufmann, San Mateo, CA, pp. 301-306, 1988.

[5] K.D. Forbus. "Qualitative Process Theory", *Artificial Intelligence*, vol. 24, pp. 85-168, 1984.

[6] D.L. Hibler and G. Biswas. "The Thought Experiment Approach to Qualitative Physics", *Proc. IJCAI-89*, Morgan Kaufmann, San Mateo, CA, pp. 1279-1284, 1989.

[7] D.L. Hibler and G. Biswas. "TEPS: Applying the Thought Experiment Methodology to Qualitative Problem Solving", to appear, *Recent Advances in Qualitative Physics*, B. Faltings and P. Struss, eds., MIT Press, 1991.

[8] D.L. Hibler and G. Biswas. "Restriction of Qualitative Models to ensure Specific Behavior in TEPS", *Tech. Report CS 90-17*, Dept. of Computer Science, Vanderbilt University, Nashville, TN, December 1990.

[9] B. Kuipers. "Qualitative Simulation", *Artificial Intelligence*, vol. 29, pp. 289-388, 1986.

[10] B. Kuipers. "Taming Intractable Branching in Qualitative Simulation", *Proc. IJCAI-87*, pp. 1079-1085, 1987.

[11] D.S. Weld and .J. de Kleer, eds. *Readings in Qualitative Reasoning about Physical Systems*, Morgan Kaufmann, San Mateo, CA, 1989.

[12] B.C. Williams. "Qualitative Analysis of MOS Circuits", *Artificial Intelligence*, vol. 24, pp. 281-346, 1984.