A Constructive Approach to Qualitative Fuzzy Simulation

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ABSTRACT. This paper presents an alternative approach to qualitative fuzzy simulation. *QFSIM* is based on extending the conventional numeric Euler's method so that it can handle qualitative coefficients, represented by fuzzy numbers. This way, time is taken as an external variable which remains unaffected by inherent inaccuracy. The time step size is a constant parameter of the simulation. These options make synchronization with pure numerical simulations or real sampled observations much easier. As the simulation provides the possible instantaneous values of the variables, the procedure *QBG* is then presented to generate their possible global qualitative behaviors on a given time horizon.

QR'92, 6th International Workshop on Qualitative Reasoning about Physical Systems Edinburgh, 24-27 July 1992.

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** Part of this work was performed as the author was on leave at the
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1. Introduction

In this paper we present a new approach for qualitative fuzzy simulation (*QFSIM*) inspired by numerical simulation methods. Current qualitative simulation methods [FOU-90] are based on constraint propagation. They generate all possible states and use filtering techniques to validate these states. A recent direction has been to add numerical information and take advantage of it in the filtering process, the basic algorithm remaining the same [BER-90][SHE-90].

An alternative approach, which has never been investigated so far, is to extend conventional numerical methods in such a way that they can handle equations with inaccurate coefficients. Within our approach, inaccuracy is captured by variables and coefficients represented by fuzzy numbers, inducing that the results of the simulation are expressed in the fuzzy formalism as well. This type of simulation relies on a 'constructive' algorithm in the sense that the new state value is calculated from the last value. Compared to qualitative simulation with added numerical data [BER-90][SHE-90], it presents the advantage to be synchronised on a precise time scale. The time step size is constant and precisely defined like in conventional numeric simulations and the inaccuracy of the model only affects the scale of variable values. This provides a firmer ground for comparing the results of the simulation with real observations, which is crucial in supervisory systems we are interested in.

Our first attempt was to use Euler's method extended to conventional fuzzy operators based on the Extension Principle [ZAD-65]. This simulation produced too much spurious behaviors, mainly because of the strong interactivity among the variables [VES-91a,b].

The two methods proposed in this paper, namely the Extremity Method and the Discretization Method, produce better results. The first one is complete but not sound, eventhough it produces much less spurious results than the one mentioned before. On the other hand, the second one is sound and it converges towards completness as the discretization is refined. Soundness and completness are understood with respect to variables instantaneous values. Indeed, these methods specify the possible values of the variables at each instant. Global behaviors of each variable (sequences of qualitative states on a given simulation window) are not directly available from the simulation. A procedure *(QBG)* based on the Discretization Method is presented in the last section to generate the set of global qualitative behaviors from the results of the simulation. Qualitative states are defined over fuzzy time intervals.

QFSIM and *QBG* have been prototyped in Common-Lisp on a Sun workstation. The man-machine interface is realised with Suntools and the simulated curves are drawn on an Apple Macintosh with Works.

2. Preliminaries

The Qualitative Fuzzy Quantity Space

The variables and coefficients of the equations are assumed to take their values in a **Qualitative Fuzzy Quantity Space** *(QFQS)* [SHE-90]. A **qualitative fuzzy value** *(qfv)* is defined as a 5-tuple [Q, a, b, α, β] (see Figure 2.1), where Q is the symbol associated to the *qfv* and (a b α β) the fuzzy quantity M with membership function $\mu M(u)$ defined as : (see [DUB-80])

$\mu M(u) = 1$ if $a \leq u \leq b$
$\mu M(u) = 0$ if $u \leq (a - \alpha)$ or $u \geq (b + \beta)$
$\mu M(u) = \alpha^{-1} (u - a + \alpha)$ if $a - \alpha < u < a$
$\mu M(u) = \beta^{-1} (-u + b + \beta)$ if $b < u < b + \beta$
where $(a, b) \in R$, $a \leq b$, $\alpha, \beta \geq 0$.

The 4 values a, b, a - α et b + ß of a fuzzy quantity (a b α ß) are called the *"extremities"*. The *QFQS* is a set of *qfv* corresponding to a finite discretization of the whole numeric range of interest of the variable. Such representation of qualitative values extends interval representations by including the idea of "preference". Smooth boundaries are usefull in capturing our commom sense intuition (see discussions in [SHE-90]). The simulation is performed with the fuzzy quantities of the coefficients. The fuzzy results of the simulation are then brought back to the *QFQS* by using the Approximation Principle [YAG-87]. This principle relies on determining the set of *qfv's* which intersect the fuzzy value. If the intersection is more than one *qfv*, distances between the fuzzy value and each *qfv* are calculated. The closest *qfv* is selected.

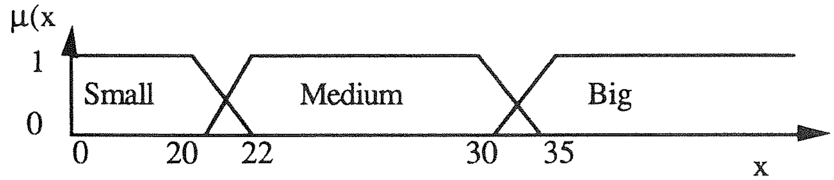
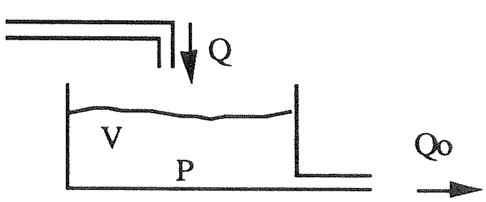


Fig. 2.1 - A Qualitative Fuzzy Quantity Space.

The Reduction

Following the Compositional Principle [DEK-84], our approach starts from the model of the components defined by a zero or first order differential equation with coefficients taking values in a *QFQS*. The Static Set contains the zero order equations and the Dynamic Set contains the first order equations (in canonical form [IWA-88]). These sets together correspond to a self-contained mixed structure [IWA-88].

The Reduction Procedure transforms the mixed structure in a single qualitative differential equation *(qde)*. An example of reduction is shown below :



Q - Input Flow
Qo - Output Flow
P - Pressure
V - Volume

Mixed Structure

Q - Qo = dV/dt
P = k1 . V
Qo = k2 . P

Reduction
==========>

Single Differential Equation *(qde)*

dV/dt + k . V = Q
with k = k1 . k2

Fig. 2.2 - Reduction of the mixed structure representing the bathtub.

The fuzzy value of the *qde* coefficients are determined by using fuzzy arithmetic operations [ZAD-65]. In the bathtub example above, k = k1 . k2, where . is the fuzzy multiplication. At this stage, it is assumed that the coefficients appearing in the mixed structure do not interact. Hence the unaccuracy captured in the mixed structure is entirely preserved in the *qde*.

3. Qualitative Fuzzy Simulation - *QFSIM*

The goal of *QFSIM* is to determine the *qfv* of the variables at each instant. Two methods for a constructive fuzzy simulation algorithm are presented : the Extremity Method and the Discretization Method. Both apply to first and second order *qde's* [VES-91a,b]. Because of space constraints, the Extremity Method will be applied to a first order system and the Discretization Method to a second order system.

3.1. The Extremity Method

In our first attempt to *QFSIM* the Euler's method was extended to conventional fuzzy operators. These fuzzy operations accord to the Extension Principle [ZAD-65] for non-interactive variables, i.e. : $\forall$ w, $\mu f_{M,N}$ (w) = sup {min (μM(u), μN(v)) : w = f(u, v)}. As it could be foreseen, this simulation produced too much spurious behaviors, being complete but not sound. The main reason is the strong interactivity among the variables which comes with The Extended Extension Principle framework, i.e. $\forall$ w, $\mu' f_{M,N,D}$ (w) = sup {min (μM(u), μN(v)) : w = f(u, v), (u, v) $\in$ D}, where D specifies the relation between u and v.

The Extremity Method precisely focuses on the interactivity among the variables. For example, in the first order system dX/dt + kX = f(t), there is a strong relation between X and its derivative dX/dt (dX/dt = f(t) - kX). As these variables take part in the same operation X(t + dt) = X(t) + dt . dX/dt(t) in Euler's method, undesirable side effects are encountered.

The Extremity Method uses the extremities a, b, a - α et b + ß of a fuzzy quantity (a b α ß). The objective is to garantee that at each simulation step the possible values of the variable are within the calculated extremities. In other words, the method must be complete, producing the least number of spurious values at each time point.

3.1.1. Application to First Order Systems

The Extremity Method applied to the first order system dX/dt + k.X = f(t), where k belongs to the fuzzy quantity K = (a b α ß), is presented below. The method successively calculates $X(t_0+dt)$, $X(t_0+2dt)$, etc. The initial values t_0 and $X(t_0)$ and the time step dt are given. The following illustrates how the method provides X(t') = (xm' xM' γ' δ') with t' = t+dt from X(t) = (xm xM γ δ).

(1) Calculate the intervals X1, X2, X3 and X4 with the extremities of X(t), i.e. xm, xM, (xm - γ) and (xM + δ) respectively.

Let us set X(t) = (xm xM γ δ), K = (a b α ß), f(t) = (fm fM fα fß) and dt be a real number. The arithmetic operators +, - and . are fuzzy operators based on the extension principle.

X1 = (x1m x1M 0 0)
= (xm xm 0 0) + [dt . [(fm fM 0 0) - [(a b 0 0) . (xm xm 0 0)]]]
X2 = (x2m x2M 0 0)
= (xM xM 0 0) + [dt . [(fm fM 0 0) - [(a b 0 0) . (xM xM 0 0)]]]
X3 = (x3m x3M 0 0)
= (xm-γ xm-γ 0 0) + [dt . [(fm-fα fM+fß 0 0) - [(a-α b+ß 0 0) . (xm-γ xm-γ 0 0)]]]
X4 = (x4m x4M 0 0)
= (xM+δ xM+δ 0 0) + [dt . [(fm-fα fM+fß 0 0) - [(a-α b+ß 0 0) . (xM+δ xM+δ 0 0)]]]

Figure 3.1 shows a simple case in which the intervals X1, X2, X3 and X4 do not intersect.

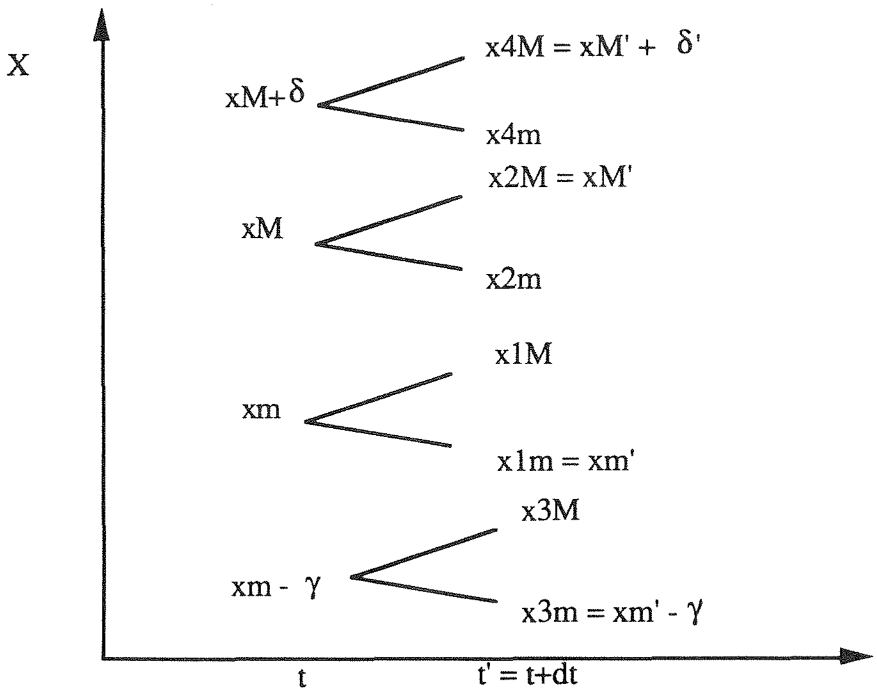


Fig. 3.1 - The Extremity Method.

(2) Compose the fuzzy quantity $X(t') = X(t + dt) = (xm'\ xM'\ \gamma'\ \delta')$:

$xm' = \min(x1m, x2m)$
$xM' = \max(x1M, x2M)$
$\gamma' = xm' - \min(x3m, x4m)$
$\delta' = \max(x3M, x4M) - xM'$

3.1.2. Example and Discussion

Figure 3.2 shows the application of the method to the first order system with K = (0.2 0.3 0.1 0.1) and f(t) = 10 if $0 \leq t \leq 14$, -10 if $t > 14$.

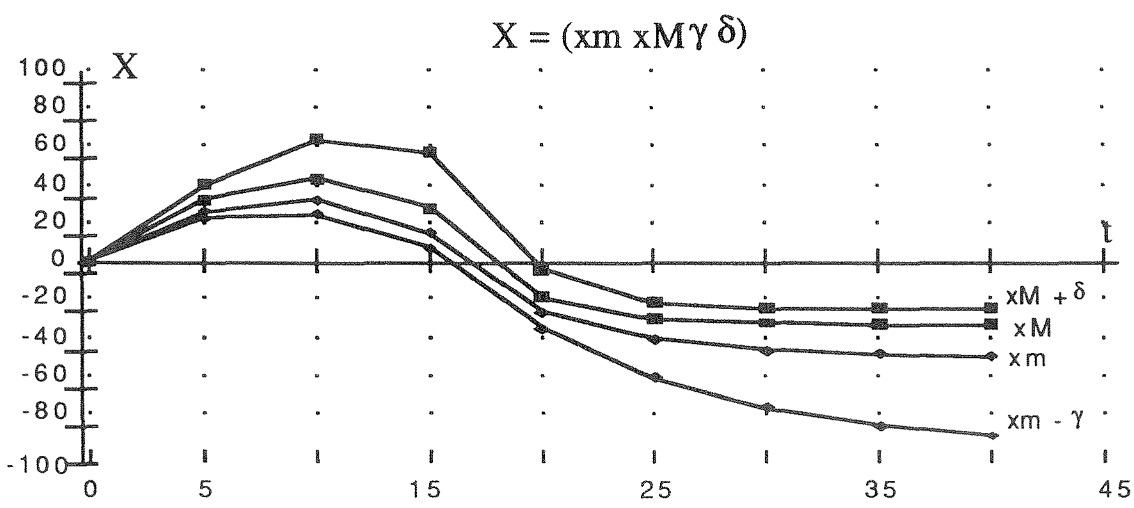


Fig. 3.2 - Fuzzy simulation with the Extremity Method.

It was proved in [VES-91a,b] that the Extremity Method applied to the first and second order systems is complete. Although the method is not sound, it produces much better results than the fuzzy extended Euler's method. The main drawback of the method is the difficulty to be generalized to more complex systems in which the relations between the variables and their derivatives are more complicated.

3.2. The Discretization Method

The Discretization Method consists in discretizing the fuzzy quantities into a set of real numbers, then successively apply the current Euler's method with every of these numbers (or with every two-per-two combination of discretised values when dealing with second order systems). The results of these simulations are used in a second step to compose the fuzzy solution.

3.2.1. Application to Second Order Systems

In this section we present the Discretization Method applied to the second order system $d^2X/dt^2 + k.dX/dt + j.X = f(t)$, where k belongs to the fuzzy quantity $K = (a\ b\ \alpha\ ß)$ and j belongs to the fuzzy quantity $J = (c\ d\ \varphi\ \psi)$.

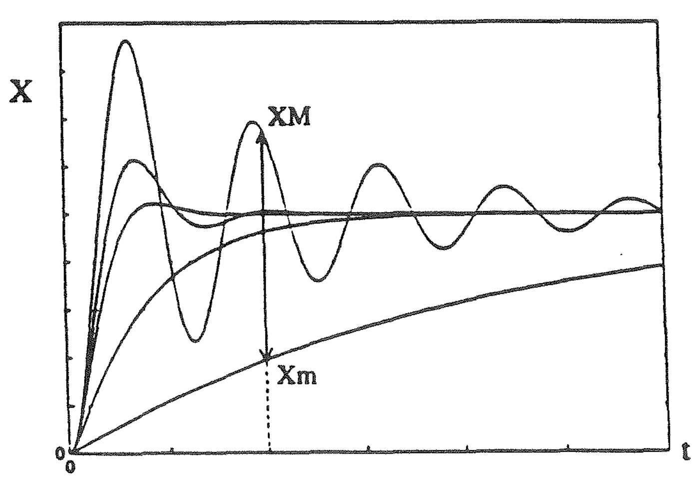
(1) Discretise the fuzzy quantities K and J and generate the two following sets of real pairs, called the *discretization sets*. The first set KJ1 contains pairs (k1,j1) such that membership functions $\mu K(k1) = 1$ and $\mu J(j1) = 1$. The second set KJ0 contains pairs (k0,j0) such that membership functions $\mu K(k0) > 0$ and $\mu J(j0) > 0$.

$K = (a\ b\ \alpha\ ß)$ and $J = (c\ d\ \varphi\ \psi)$
$KJ1 \subseteq \{(k1,j1) / k1,j1 \in R,\ a \le k1 \le b,\ c \le j1 \le d\}$
$KJ0 \subseteq \{(k0,j0) / k0,j0 \in R,\ (a - \alpha) \le k0 \le (b + ß),\ (c - \varphi) \le j0 \le (d + \psi)\}$

Obviously $KJ1 \subseteq KJ0$. By sick of clarity, the details for determining the discretization sets are provided in Section 3.2.2.

(2) Apply Euler's method to each pair (k_i, j_i) that belongs to KJ0:

for all $(k_i, j_i) \in KJ0$, $dx/dt(t, k_i, j_i) = v(t, k_i, j_i)$
$x(to, k_i, j_i) = X(to)$
$v(to, k_i, j_i) = dX/dt(to)$
$x(t+dt, k_i, j_i) = x(t, k_i, j_i) + dt \, . \, v(t, k_i, j_i)$
$v(t+dt, k_i, j_i) = v(t, k_i, j_i) + dt \, .[\, f(t) - j_i \, . \, x(t, k_i, j_i) - k_i \, . \, v(t, k_i, j_i) \,]$



$Xm = \min x(t,k1,j1),\ (k1,j1) \in KJ1$
$XM = \max x(t,k1,j1),\ (k1,j1) \in KJ1$

Fig. 3.3 - Fuzzy simulation with the Discretisation Method.

(3) Compose the fuzzy solution X :

$X(t) = (Xm\ XM\ \gamma\ \delta)$

$Xm = \min x(t, k1, j1), (k1, j1) \in KJ1$

$XM = \max x(t, k1, j1), (k1, j1) \in KJ1$

$\gamma = Xm - \min x(t, k0, j0), (k0, j0) \in KJ0$

$\delta = \max x(t, k0, j0) - XM, (k0, j0) \in KJ0$

3.2.2. The Discretization Procedure

It was proved in [VES-91a,b] that the Discretization Method is sound and converges towards completness as the discretization is refined. The method is not complete in practical finite discretization cases but the particular way of discretizing may significantly influence the results. This section presents a procedure to discretise the fuzzy coefficients in a constructive way.

Consider the second order system $d^2X/dt^2 + k.dX/dt + j.X = f(t)$, where k belongs to the fuzzy quantity $K = (a\ b\ \alpha\ \beta)$ and j belongs to the fuzzy quantity $J = (c\ d\ \varphi\ \psi)$, then the qualitative nature of the solutions is determined by the roots λ1 and λ2 of the characteristic equation $\lambda^2 + k\lambda + j = 0$. Assuming that $j \neq 0$, eight cases must be considered : (see Figure 3.4)

A : λ1 and λ2 are complex on the right half-plane ($\sigma > 0$)

B : λ1 and λ2 are complex on the axis jw ($\sigma = 0$)

C : λ1 and λ2 are complex on the left half-plane ($\sigma < 0$)

D : λ1 = λ2 ($\Delta = [k^2 - 4j]^{1/2} = 0$) is a real on the left half-plane ($\sigma < 0$)

E : λ1 and λ2 are reals on the left half-plane ($\sigma < 0$)

F : λ1 and λ2 are reals, λ1 on the right half-plane and λ2 on the left half-plane

G : λ1 and λ2 are reals on the right half-plane ($\sigma > 0$)

H : λ1 = λ2 ($\Delta = [k^2 - 4j]^{1/2} = 0$) is a real on the right half-plane ($\sigma > 0$)

Figure 3.4 shows the phase space analysis of cases B, C, E and F, corresponding to different qualitative behaviors for the variable X.

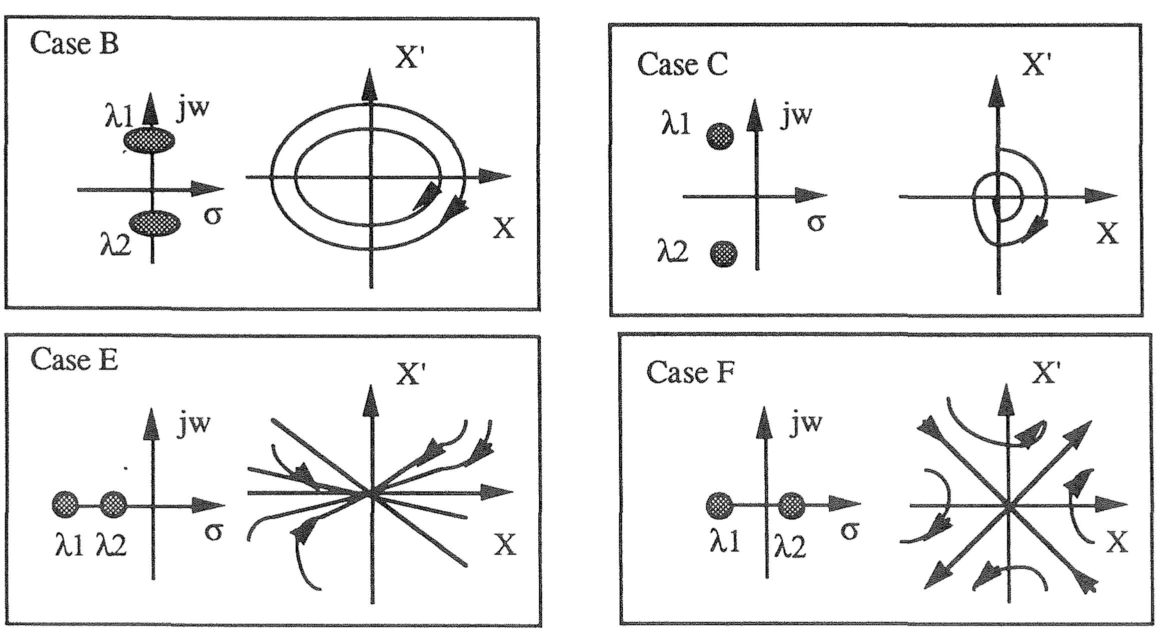


Fig. 3.4 - Phase space analysis of cases B,C,E and F.

Figure 3.5 shows the eight regions A to H in the plane j x k (k is the horizontal axis and j is the vertical axis). The curves ($\Delta = k^2 - 4j = 0$), ($j = 0$) and ($k = 0, j > 0$) delimit the different regions. Since the coefficients j and k take their values ($\mu > 0$) in the intervals $[a - \alpha, b + \beta]$ and $[c - \varphi, d + \psi]$ respectively, different qualitative behaviors are possible depending on the regions intersected by these intervals. For example, if $(a - \alpha)$ and $(c - \varphi)$ are negative, and if $(b + \beta)$ and $(d + \psi)$ are positive, all cases A to H are possible.

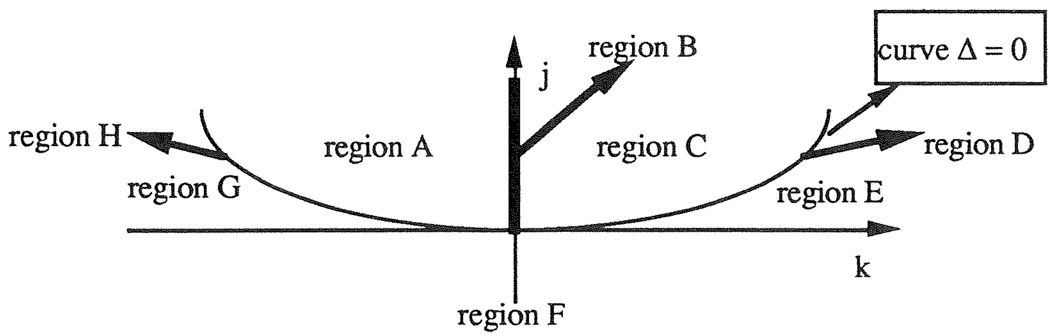


Fig. 3.5 - Regions A to H in the plane j x k.

The Discretization Procedure chooses the pairs (k, j) from the intervals $[a - \alpha, b + \beta]$ and $[c - \varphi, d + \psi]$ in the following way :

(1) It garantees that any possible qualitative behavior will be simulated at least once.

(2) It chooses the pairs (k, j) according to the surface intersecting each of the regions A to H. The number of pairs (k, j) is proportional to the surface.

For example, considering the case in which $(a - \alpha)$, $(c - \varphi)$, $(b + \beta)$ and $(d + \psi)$ are all positive, then the whole surface $S = [(b + \beta) - (a - \alpha)] . [(d + \psi) - (c - \varphi)]$ is in the right upper quadrant and it intersects regions C, D and E. If N is the desired number of discretizations, the procedure selects :

- N_E pairs (k, j) in region E,
  where $N_E = ((N - 1) . \text{Surface\_in\_E}) / S$
  $\text{Surface\_in\_E} = 1/12 [(b + \beta)^3 - (a - \alpha)^3] - [(c - \varphi) . (b + \beta - a + \alpha)]$;

- $N_D = 1$ pair (k, j) in region D;

- N_C pairs (k, j) in region C,
  where $N_C = ((N - 1) . \text{Surface\_in\_C}) / S$
  $\text{Surface\_in\_C} = [(d + \psi) . (b + \beta - a + \alpha)] - 1/12 [(b + \beta)^3 - (a - \alpha)^3]$.

The Discretisation Procedure determines which regions are intersected, calculates the number of pairs (k, j) in each region and selects the pairs. Additionally, two heuristics are used by the procedure to select the pairs :

(1) take frontier points;
(2) take the points as much spread as possible.

Let us notice that in case of first order systems the Discretization Procedure is restricted to selecting the values of one single coefficient as much spread as possible.

4. Qualitative Behaviors Generation

The Qualitative Fuzzy Simulation *QFSIM* determines the *qfv* of the variables at each instant. The fuzzy value represents the possible ($\mu > 0$) and the really possible ($\mu = 1$) values of the variable. Now, should we aim at providing the possible *qualitative behaviors* of the variables as well, i.e. the sequences of different qualitative states, that *QFSIM* is not sufficient. If we consider the example in Figure 4.1, it could be concluded that the sequence BIG-MEDIUM-BIG between t = 10 and t = 12 is a possible behavior for variable X. Indeed X(10) = BIG, X(11) = MEDIUM and X(12) = BIG are possible values of X. However, there is no real solution corresponding to this qualitative behavior.

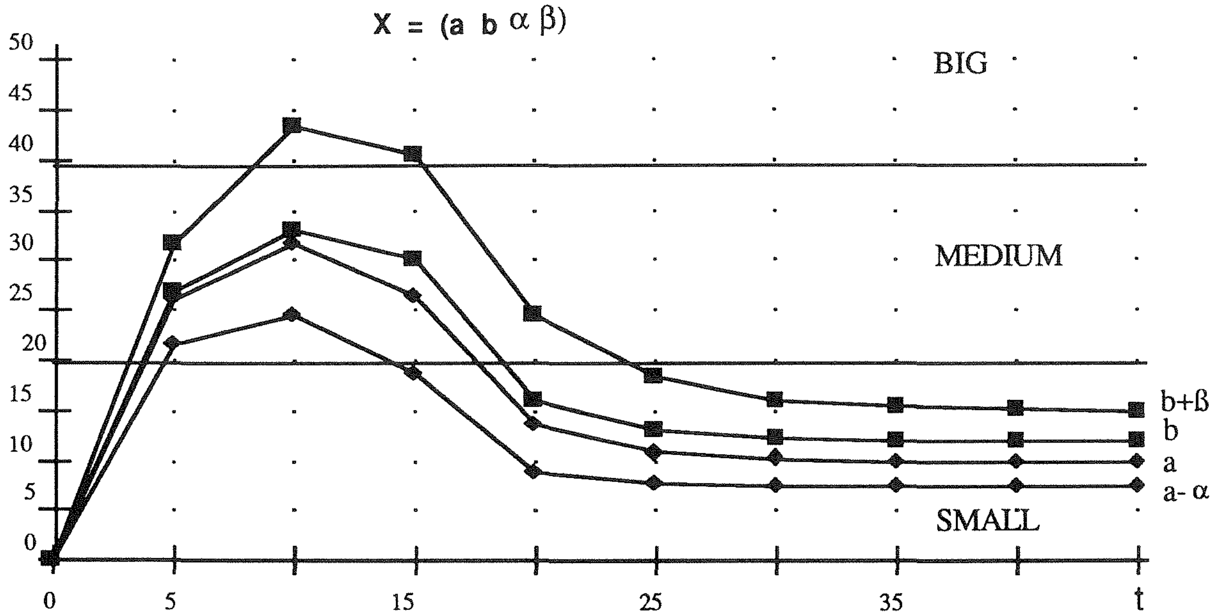


Fig. 4.1 - A fuzzy simulation.

The set of possible qualitative behaviors is shown in Figure 4.2. The meaning of Iµ1 and Iµ0 will be specified later.

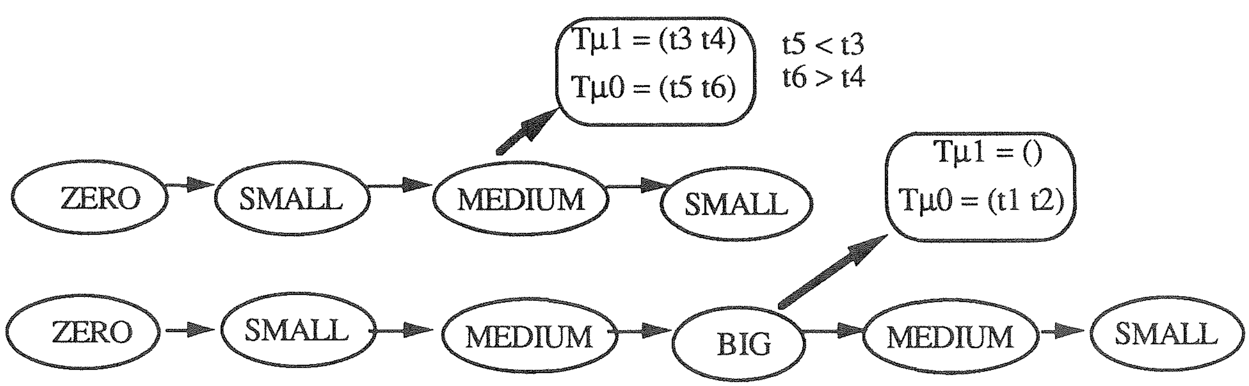


Fig. 4.2 - The possible qualitative behaviors.

These are the results provided by the "Qualitative Behavior Generator" *QBG* when applied to the first order system dX/dt + k.X = f(t), where k is the *qfv* [K, a, b, α, ß].

4.1. The Qualitative Behavior Generator *QBG*

This section presents the detailed description of the method used by *QBG*. It is based on the Discretization Method presented in Section 3.2. For sick of clarity, the method is presented for first order systems (one single coefficient k) but *QBG* can handle second order systems as well.

(1) Consider the results of the Discretization Method and for all k_i in the discretization set $K0 \subseteq \{k0 / k0 \in R, (a - \alpha) \leq k0 \leq (b + \beta)\}$, build the qualitative behavior $QB(k_i)$ of variable X.

Notations :

$QB(k_i)$ = Sequence of QS	: Qualitative Behavior of X, calculated with the real coefficient k_i
QS = (QD, $T\mu0_i$, $T\mu1_i$)	: Qualitative State
QD	: Qualitative Descriptor
$Q(x(t, k_i))$	: Qualitative value (or description) of $x(t, k_i)$ at instant t
$T\mu0_i = [t1, t2]$	: Maximal time interval for which the QS is possible (μ>0)
$T\mu1_i = [t1, t2]$	: Maximal time interval for which the QS is really possible (μ=1)

For each k_i the procedure proceeds to a real simulation for *initial time < t < horizon*, determining $x(t, k_i)$. Qualitative values $Q(x(t, k_i))$ are derived from the real values $x(t, k_i)$ using the approximation principle presented in Section 2. The procedure then uses these values $Q(x(t, k_i))$ to compose $QB(k_i)$. There is a change of qualitative state each time that the QD changes. The algorithm garantees qualitative continuity [VES-91a].

Two time intervals $T\mu0_i$ and $T\mu1_i$ are associated to each QS in $QB(k_i)$. A QS is possible (μ>0) during the interval $T\mu0_i$ and is really possible (μ=1) during the interval $T\mu1_i$. Given T the connex time interval during which some QD holds in a particular $QB(k_i)$, $T\mu0_i$ and $T\mu1_i$ are calculated in the following way : if $k_i \in K1$ then $T\mu0_i = T\mu1_i = T$, otherwise, if $k_i \in K0 - K1$ then $T\mu0_i = T$ and $T\mu1_i = \varnothing$. These intervals $T\mu0_i$ and $T\mu1_i$ calculated for each QS of a particular $QB(k_i)$ will then be used in step (3) to calculate the intervals $T\mu0$ and $T\mu1$ of each QS of a QB of variable X.

(2) Build the set SQB of qualitative behaviors of variable X.

Notations :

SQB	: Set of QBs
QB	: Qualitative Behavior (Sequence of QS)

Each $QB(k_i)$ is characterized by a sequence of qualitative descriptors (QD) with associated possible and really possible time intervals. There will be as much QBs in SQB as there are different sequences of descriptors. All the $QB(k_i)$s having the same sequence of descriptors are composed in the same QB. This is illustrated by the example below with two $QB(k_i)$s :

$$QB(k_1) = ((QSD^1, T\mu0_1{}^1, T\mu1_1{}^1)(QSD^2, T\mu0_1{}^2, T\mu1_1{}^2))$$
$$QB(k_2) = ((QSD^1, T\mu0_2{}^1, T\mu1_2{}^1)(QSD^2, T\mu0_2{}^2, T\mu1_2{}^2))$$

The QB resulting from the composition of $QB(k_1)$ and $QB(k_2)$ is :

$$QB = ((QSD^1, T\mu0^1, T\mu1^1)(QSD^2, T\mu0^2, T\mu1^2))$$

where $T\mu0^j = \bigcup_i T\mu0_i^j$, $T\mu1^j = \bigcup_i T\mu1_i^j$, for i,j = 1, 2 in our example.

It is important to notice that the Discretization Procedure as described in Section 3.2.2. guarantees that *QBG* will detect *all* the qualitatively different behaviors (oscillations, asymptotical trend, ...). In this meaning, *QBG* is complete and sound.

4.2. Example

In Figure 4.1 illustrates the example of a qualitative fuzzy simulation of the first order system with coefficient k = [SMALL, 0.2, 0.3, 0, 0] and f(t) = 10 if $0 \le t \le 14$ and f(t) = -10 if $t > 14$.

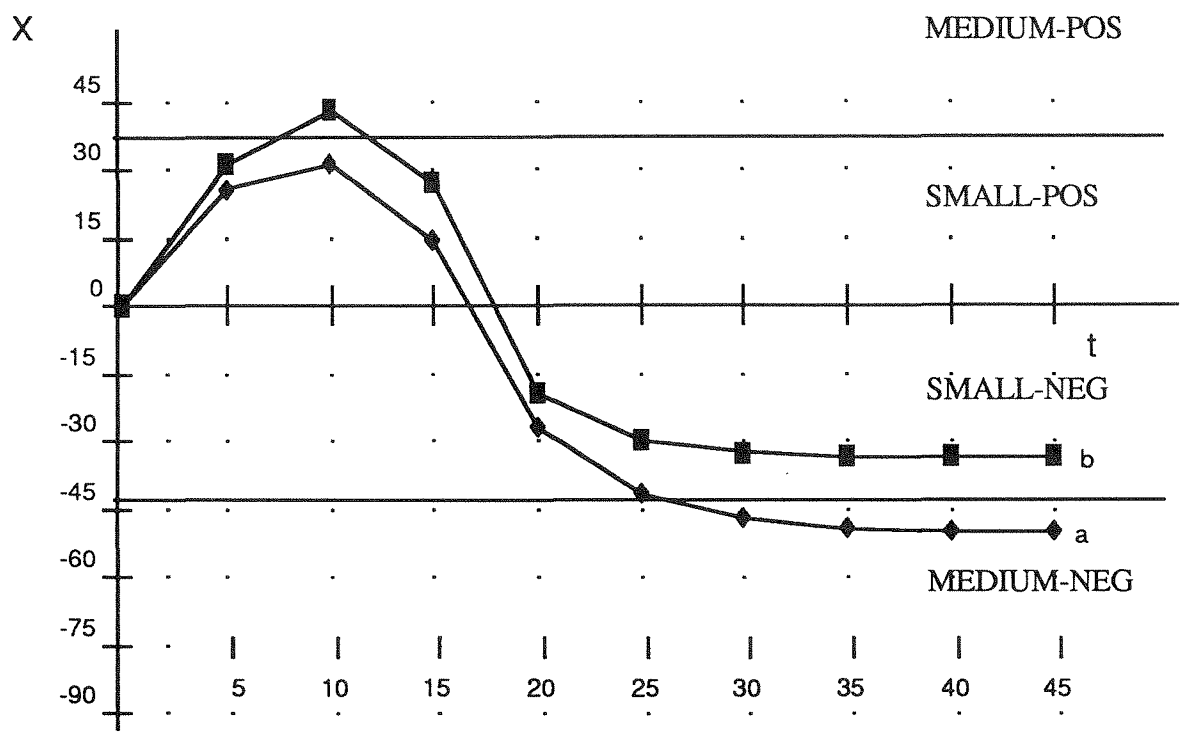


Fig. 4.3 - A qualitative fuzzy simulation of a first order system.

QBG derived the following two possible qualitative behaviors :

Behavior 1

	ZERO ➡	**SMALL-POS** ➡	**ZERO** ➡	**SMALL-NEG** ■
Tμ0	(0 0)	(1 16)	((15 15)(16 16))	(15 45)

Behavior 2

	ZERO ➡	**SMALL-POS** ➡	**MEDIUM-POS** ➡	**SMALL-POS**
Tμ0	(0 0)	(1 11)	(8 13)	(14 15)
	➡ **ZERO** ➡	**SMALL-NEG** ➡	**MEDIUM-NEG** ■	
Tμ0	(16 16)	(16 27)	(24 45)	

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5. Conclusions and future work

The work that we have presented in this paper is used in a dynamic process supervisory system. It shows two complementary aspects, appearing as two separate procedures, which provide complementary answers to supervision system requirements. Given the model of the process in which the parameters are not accurately known, the two following issues have been addressed :

(1) Predicting the values that the variables may take at each instant of a given temporal window, the scale for the variable *time* remaining precisely defined. *QFSIM* provides two simulation methods for which variable intantaneous values are obtained in the form of fuzzy sets which exibit *possible values* and *really possible values*, depending on whether the membership function is equal or inferior to 1.

(2) Predicting the qualitative behavior of the variables in terms of a sequence of qualitative descriptors that have been appropriately defined. The qualitative reasoner *QBG* uses the Discretisation Method to generate all the qualitative behaviors. Temporal information is also available since every qualitative descriptor in the sequence has associated two fuzzy time intervals, indicating the *possible* and *really possible* state duration as well as the initial and final time points.

In our opinion, these two aspects are as important and definitely complementary. It is enough to take the example of model-based diagnostic systems based on the comparison of predictions and observations. Indeed it may be as important to be able to compare instantaneous values on several precise reference time points as trajectory qualitative shapes on a given temporal window and qualitative state changes.

The type of simulation presented in this paper uses synchronised sampling, in accordance with most industrial process monitoring systems which proceed themselves to sampled observations. Tracking the process is thus significantly facilitated. The time sample rate is a constant parameter to be chosen like in conventional numeric simulations. Then the inaccuracy of parameters in the model only affects variable values estimates. That provides a firmer ground for again comparing the results of the simulation with real observations, which is crucial in real time supervisory systems. In non constructive qualitative simulation algorithms, temporal durations are calculated with the first order Taylor-Lagrange formulae using quantity space values in the form of numeric or fuzzy intervals. It was shown in [MIS-90] that the first order Taylor-Lagrange formulae is scarcely sufficient to provide significant information. This is true, independently of the weakness directly related to a weak quantity space, at the neighbourhood of critical points for which dx/dt reaches zero. Indeed, zero derivative leads to one *infinite* boundary for the duration estimate. As a result, time durations calculated for adjacent states are often widely overlapped. It may happen that a given instant belongs to several consecutive duration estimates, implying that the value of the variable at this instant is very weakly constrained. In this cases, instantaneous value comparison is mostly unefficient.

On the other hand, we are aware that the counterpart of this type of approach is to require much more calculations than a qualitative reasoner would need to infer the qualitative state changes. In this aspect it is closer to numerical simulation algorithms.

Finally, it may be interesting to discuss the completness and soundness issue. Other approaches are unable to guaranty soundness. Indeed, they use constraint propagation with interval labels which is complete but not sound. Consequently, spurious behaviors may be generated. The same happens with our Extremity Method. The Discretisation Method which relies on running several well-chosen numerical simulations is sound, but it is in turn not complete. However, it is possible to provide a *measure of completness* as it converges towards completeness as the discretization granularity increases. The complexity of the method is $N^{(n+1)}$, where N is the number of discretizations and n is the order of the system. Since the discretization is performed in a rational and constructive way, *QBG* is itself complete in the sense that it predicts all the qualitatively different behaviors (oscillations, asymptotical trend, ...).

FQSIM is presently used to simulate the behavior of a steel process at *CST ("Companhia Siderurgica de Tubarao")* which is a Brazilian-Japanese company located in Vitoria (Brazil). It consist of three subsystems in cascade: one is a second order system and the other two are first order piecewise linear systems with delays. At this point the results are rather encouraging. However, significant work remains to be done before *QFSIM* and *QBG* can deal with real complexity problems. An important step in this direction is to extend the algorithms so that they can deal with non linear systems. We have been analysing non linear systems which can be approximated by *piecewise linear* ones. Further investigations may consider the idea to use the Discretization Method by taking advantage of the work by Sacks [SAC-90]. Indeed, [SAC-90] presents the system *PLR* (Piecewise Linear Reasoner) which is able, applying theoretical methods issued from system theory, to produce a qualitative description of the solutions for all initial values of parametrized ordinary differential equations. *PLR* indicates the types of the solutions (phase space trajectory shapes), giving important information about the variables behavior but it does not generate these behaviors. We intend to have *QFSIM* generating these behaviors with a rational and constructive discretization procedure based on *PLR*'s results.

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