Modelling the Influence of Non-Changing Quantities

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Abstract

In qualitative modelling, information is lost by abstracting from quantitative formulae. We show that when the behaviour of two similar systems is compared, non-changing quantities from these formulae can have a significant influence on the qualitative prediction. We propose the addition of a new ontological primitive for representing these influences in qualitative models, and provide a calculus for exploiting this primitive in the reasoning process. Augmentation with the new primitive enhances the power of the qualitative simulator, resulting in a more appropriate prediction of behaviour, and also improves the explanation capacities of the model. The latter feature is of major importance for tutoring systems using qualitative reasoning.

1 Introduction

A behaviour description generated by qualitative simulation consists of a set of states modelling qualitatively distinct behaviours of the simulated device. The notion of change is the key concept for generating such a description. In qualitative reasoning basically, two ways of dealing with changes have been investigated: values and (in-)equalities. In particular in the early days of qualitative reasoning research, distinct states of behaviour were defined as having different values for quantities and/or different values for their derivatives. Finding a state transition implied searching for a quantity whose derivative was *plus* or *minus* so that it would adopt a higher or lower value in its quantity space [3, 6]. In later publications, reasoning with (in-)equalities became more important (see for instance [9]). Looking for state transitions became more complex and included reasoning steps such as

$$\textbf{if } A = B \textbf{ and } \Delta A = \textit{plus} \textbf{ and } (\Delta B = 0 \textbf{ or } \Delta B = \textit{minus}) \textbf{ then } A > B.$$

In the approach we are using, both techniques for reasoning with changes can be employed (cf. [1]). Still we had severe trouble in modelling the behaviour of the balance system (see Figure 1)[1]. Certain changes in the behaviour of this system cannot be represented adequately. This

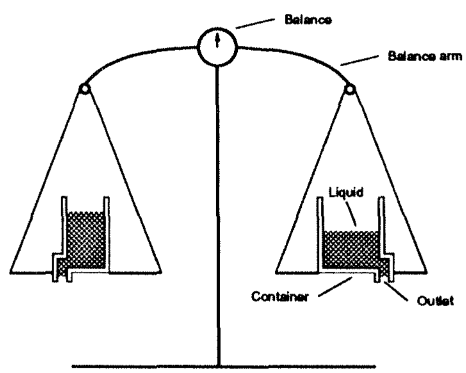

Figure 1: A Balance Problem

behaviour depends on 'influences' of quantities that don't change themselves, but still have a significant effect on how the behaviour of the system evolves. For example, if the heights of the water columns in both containers are equal (and therefore also the pressures at the bottom and thus the flow rates) then the containers will lose water with equal rates. If the widths of the containers are also equal, both containers will be empty at the same time and the height of the water columns will stay equal while emptying. However, a problem occurs if the widths of the containers are unequal. In that case, the container with the smallest water column will become empty first. In order to reach that state of behaviour the heights of the columns have to become unequal first. The height of smallest column will become lower (see Figure 2, transition from State 3 to State 4). In other words, the change in height depends, among others, on the width of the column. The width has a significant effect on how the behaviour changes, even though it does not change itself. This feature cannot be modelled adequately by current

[1]The problem is to predict the behaviour of balances with containers on each balance arm. Both containers are assumed to be equal in weight. Depending on the difference in the mass of the liquid contained by the containers, one balance arm may be heavier than the other. Therefore, after releasing it from the starting position the balance may change its position. Through outlets near the bottom of the containers the liquid gradually flows out of the containers. Depending on the pressure at the bottom, the flow rates may be different. As a result, the balance may move to a different position, because the difference in weight between the two balance arms changes. Eventually, when both containers are empty, the balance will reach an equilibrium.

techniques for qualitative reasoning that use influences and proportionalities as the basis for determining changes, because the 'influence' of the non-changing quantity is not captured by these dependencies.

We present a technique that can be used for modelling the influence of non-changing quantities. After providing some background on our research in Section 2, the problem we are facing is explained in more detail (Section 3). In Section 4, we discuss how the problem can be solved by using quantitative mathematics. Section 5 presents a reformulation of the mathematically oriented solution in qualitative terms, such that it can be added to current approaches to qualitative reasoning. Finally, in Section 6 we briefly summarise our results and discuss the consequences of them for qualitative reasoning.

2 Teaching Qualitative Reasoning

It is widely recognised that for solving physics problems a careful qualitative analysis of the problem situation is essential (cf. [8, 2, 7, 5]). A typical difference in the problem solving behaviour of experts and novices is the large amount of time an expert spends on the qualitative analysis. The novice is more likely to skip this phase and start computing formulae right away. One of the problems for physics tutors is to make sure that novices start with a qualitative analysis before they select mathematical formulae. In other words, teaching qualitative reasoning is an essential step in teaching physics.

The *Balance Tutor* [4] is our first realisation of a teaching environment that coaches students in analysing the behaviour of physical devices using qualitative knowledge. The teaching environment uses an on-line qualitative simulator (GARP) that implements a problem solving capacity similar to the traditional approaches to qualitative reasoning [1]. For teaching qualitative reasoning, two requirements must be fulfilled by the simulator:

- The simulator must predict the behaviours that are manifested by the real system. This means that the simulator may not neglect any states of behaviour, but also that it may not predict any spurious (non-existing) behaviours.

- The knowledge used by the simulator for predicting the states of behaviour should facilitate explanation of why certain behaviours occur whereas others do not. This implies that the simulator must be able to provide a causal account of why the behaviour evolves in a certain direction.

The knowledge representation we use does not completely satisfy these two constraints. The shortcomings are not just weaknesses in our simulator, but result from lacking reasoning capabilities in current qualitative reasoning techniques. In the next section we will first elaborate on the knowledge representation that we use and then explain the problem of dealing with non-changing quantities in more detail.

3 Knowledge Representation

For reasoning about the balance system we use the notion of processes as the prime cause of changes (cf. [6]). The (direct) changes imposed on a system by influences are propagated by proportionalities (indirect changes). In addition to these two causal relations, corresponding values are defined between magnitudes of specific quantities. Non-causal dependencies ($<, \leq, =, \geq, >$) may be used for representing inequalities between quantities. Note that inequalities are not the same as correspondences; two quantities can be equal but still have non-corresponding qualitative values, or vice versa.

Similar to the component oriented approach [3] and the process oriented approach [6], GARP uses the notion of *model fragments* for modelling the behaviour of some real-world system. All model fragments have associated with them a set of conditions under which they are applicable, and a set of consequences that are given once their conditions hold. Typically, conditions specify required objects, inequalities between quantities, and/or specific values that quantities must have. Consequences, on the other hand, usually introduce influences and proportionalities between quantities, although inequalities can also be specified as a consequence.

In order to reason about the behaviour of the balance system, model fragments are needed for the containers containing liquid, the liquid flow out of the containers, the position of the balance (depending on the mass difference between left and right), and the movement of the balance (depending on the flow difference between left and right). In the *scenario*, or input system, the balance system is, apart from its physical structure, defined by the quantities *Height* (H_L, H_R), *Volume* (V_L, V_R), *Width* (W_L, W_R), and *Flow* (F_L, F_R). For purposes of clarity, we will not discuss the fully corresponding quantities *Amount* and *Mass* (corresponding with *Volume*), *Pressure* (corresponding with *Height*), *Position* (corresponding with $V_L - V_R$), and *Movement* (corresponding with $F_L - F_R$). Furthermore, we assume that the containers are rectangular, and for the moment ignore the depths of the containers by assuming that they are equal.

In the example shown in Figure 2, the following inequalities initially hold: $H_L > H_R$, $V_L = V_R$ and $W_L < W_R$. All quantities have the initial value *plus*. When presenting this

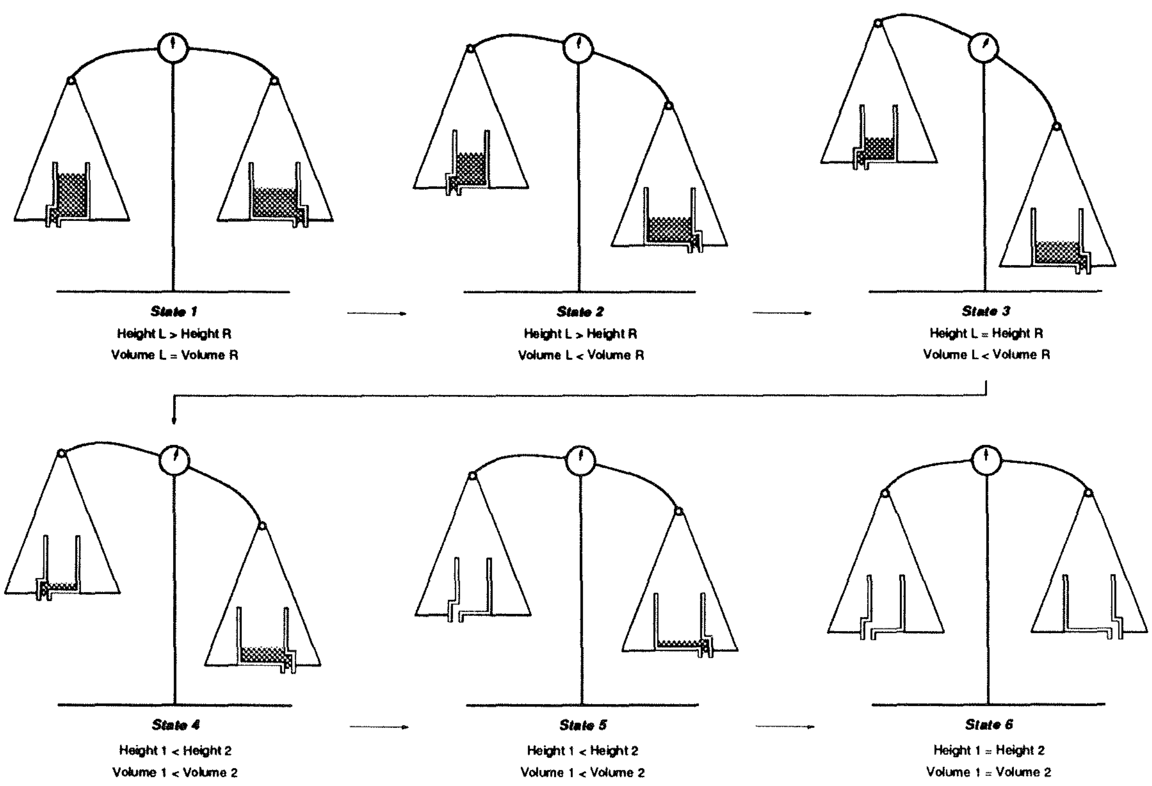

Figure 2: Behavioural Description of a Balance Problem

system to the qualitative simulator, a causal structure is produced for the different states of behaviour, as shown in Figure 3. The height of the column determines the flow rate. The flow rate influences the volume. Changes in the volume propagate into changes of the height. Changes in the width also propagate into changes in height. This causal structure is applicable for the liquid columns on both sides of the balance. However, this model of the balance system

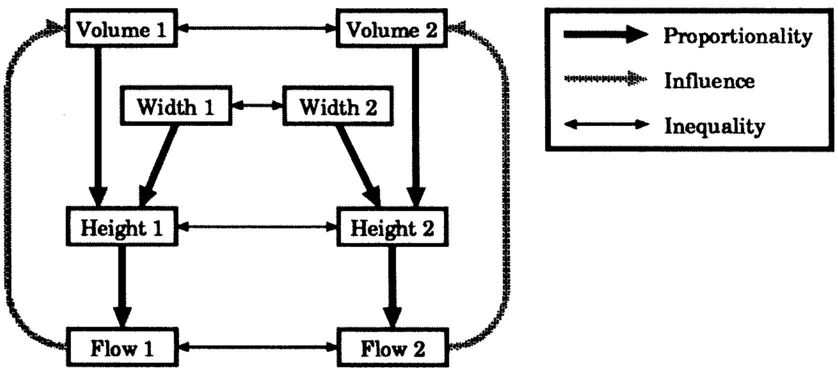

Figure 3: Causal Dependencies and Inequalities for the Balance Problem

does not allow for the derivation of all required states as shown in Figure 2. In particular the transition from State 2 to State 3 can not be derived.

The transition from State 1 to State 2 is derived because $H_L > H_R$ yields $F_L > F_R$, and consequently $\delta V_L > \delta V_R$. The volumes decrease with different rates, and will thus become unequal. The decrease rates of the heights, however, can not be derived. This is caused by the way in which the effect of the width on the liquid is represented. The height of the liquid is represented as proportional to the volume (which is decreasing) as well as to the width (which is constant). Because the relation between *Volume*, *Height* and *Width* is divided into two proportionalities, the effect of *Width* on the derivative of *Height* is lost.

The next section discusses this problem in a mathematical manner, providing the basis for introducing a new primitive relation type and its calculus.

4 Mathematical Solution

State transitions are induced by (in)equalities between the volumes V on each side of the balance system and between the heights H of the liquid columns. The relation between V and H is mediated by the area of the container. Since we assume rectangular containers with equal depths, only the width W is relevant:

$$V = W \times H \tag{1}$$

The flow rate F is defined as the decrease of the volume per second, thus $F = -\frac{d}{dt}V$. However, to avoid the confusion caused by the negative derivative, we simplify this formula to

$$F \sim \delta V \tag{2}$$

The relation between the flow and the height of the liquid column is

$$F = \alpha\sqrt{H}$$

where α is determined by various quantities that are assumed to be equal for both liquids, like the viscosity, the area of the outlet, and the gravitational constant. We therefore omit α, yielding

$$F \sim \sqrt{H} \tag{3}$$

The change of the volume is equal to the width of the liquid column times the change of its height.

$$\delta V = W \times \delta H$$

This can be rewritten as

$$\delta H = \frac{\delta V}{W} \tag{4}$$

These equations allow us to infer the state sequence that describes the behaviour of the balance. In each state, Equation 3 is used to deduce the ratio of the flow rates, and Equation 2 to deduce the ratio of the volume derivatives. Equation 4 is used to derive the ratio of the derivatives of the heights. The initial state is characterised by the inequalities $V_L = V_R$, $H_L > H_R$, and $W_L < W_R$. We describe the first three state transitions in detail. (see also Figure 2).

State 1 → State 2 We use $H_L > H_R$ to deduce $F_L > F_R$ (Equation 3), and consequently $\delta V_L > \delta V_R$ (Equation 2). Because the decreases in the volumes are unequal, their ratio changes from $V_L = V_R$ to $V_L < V_R$ in the next state.

State 2 → State 3 Since $W_L < W_R$ and $\delta V_L > \delta V_R$, we derive that $\delta H_L > \delta H_R$, using Equation 4). State 2 therefore terminates by the heights becoming equal. Another possibility would be a termination in which the left container becomes empty: because $\delta V_L > \delta V_R$, and $V_L < V_R$, we can derive that V_L may become zero. However, this requires $H_L < H_R$, and no direct transition is possible from $H_L > H_R$ to $H_L < H_R$ (the intermediate ratio $H_L = H_R$ can not be skipped).

State 3 → State 4 In State 3, $H_L = H_R$ holds, so $F_L = F_R$ (Equation 3), and thus $\delta V_L = \delta V_R$ (Equation 2). Because $W_L < W_R$ also holds, $\delta H_L > \delta H_R$ is derived using Equation 4, causing a transition to State 4.

Apparently, there are no problems in deriving the behaviour of the balance in the mathematical model. However, as observed in Section 3, the transition from State 2 to State 3 cannot be derived in the qualitative model, because some of the required information is lost in the abstraction process. A solution for this problem is presented in the next section.

5 Qualitative Solution

The mathematical solution presented in the previous section reveals the limitations of qualitative reasoning with respect to the influences of non-changing quantities. In quantitative formulae, the relations between different variables as well as constants can easily be represented. But constant values are abstracted from in building qualitative models, because they are not actively involved in the causal quantity interactions within the system, and hence do not directly influence the qualitative values of quantities. The 'hidden' constant in proportionalities and correspondences influences the *quantitative* value of the resulting quantity, however, and may become relevant in inequality reasoning, where the quantitative values of quantities are compared.

5.1 Representation

Consider again the balance example, and the representation of the quantities as given in Figure 3. The reason why the quantitative model is capable of predicting the change in the height equation while the qualitative model is not, can be attributed to the fact that in the latter the relation between *Volume* and *Height* is independent from the relation between *Volume* and

Width. Moreover, the correspondence between the width and the height will not induce any change in the height because the width has a constant value. In the qualitative model, information is lost on the interrelation between the three quantities *Width*, *Volume*, and *Height.* This loss of information can be compensated by explicitly representing that *Volume* is the product of *Width* and *Height.*

One option is to define a qualitative version of the (quantitative) multiplication relation for representing formulae in qualitative models. The formula $V = W \times H$ can now straightforwardly be represented as `mult(V,W,H)`[2]. In order to support larger multiplications (for instance, when the depth (D) is taken into account, the formula expands to $V = W \times H \times D$), the representation can be defined to allow embedded multiplication relations, or intermediate variables can be added. Incorporating the depth will then result in respectively `mult(V,mult(W,H),D)` or `mult(V,X,D), mult(X,W,H)`. We do not elaborate upon the exact representation of larger multiplications here.

Another option is to enrich the ontology for qualitative reasoning with a new primitive that we will call *relation modification.* This enables us to express explicitly that the value of a non-changing quantity affects another relation. Considering the balance example again, we can now express that the width affects the influence of the volume on the height. This may be represented as

```
prop_pos(Prop1, Volume, Height)
mod_neg(Width, Prop1)
```

The positive proportionality relation `Prop1` between *Volume* and *Height* states that an increase (decrease) in the volume causes an increase (decrease) in the height, whereas *Width* acts as a negative modifier of `Prop1`: the larger *Width*, the smaller the influence of *Volume* on *Height.* Representing more than one modifier for a proportionality is now easily represented by defining more than one `mod_pos` or `mod_neg` relation for the same proportionality.

For the purpose of incorporating the influence of non-changing quantities, there is no principal difference between these two representations. A formula $A = C \times B$, where C is a constant, can equivalently be represented as `mult(A,C,B)` or as `prop_pos(Prop1, B, A), mod_neg(C, Prop1)`. The difference is in the explicit notion of causality present in the latter representation, as is discussed in Section 6. In the remainder of this section, we adopt the latter representation for presenting the calculus.

5.2 Calculus

In designing a calculus for dealing with relation modifiers, it is important to keep in mind that these modifiers are only relevant in situations where two similar processes are compared. As long as a single causal sequence of quantity dependencies is considered (as is the case with causal prediction of behaviour for a solitary system), these modifiers can be omitted. This is exactly what happens in the current definition of a proportionality: $A = \alpha \times B$ is abstracted to $A \propto B$. When comparing two similar processes, however, it is not always possible to abstract from constant values. As soon as corresponding constant values are different, they may effect the relation between other corresponding quantities in the system. Thus, our calculus assumes the existence of two similar processes. In qualitative reasoning terms, similar processes are modelled by (sets of) instances of the same generic model fragments. This fact is exploited explicitly in our calculus.

[2]To stress the fact that multiplications are only relevant for (quantitative) inequality reasoning, the alternative representation `equal(V,mult(W,H))` could be used.

We start with the simplest case, in which there is a single proportionality with a single modifier in each of the systems. Let $A_1, B_1, \ldots$ be the quantities of one system, and $A_2, B_2, \ldots$ the corresponding quantities in the other system. Let the following relations hold[3]:

$\mathtt{prop\_pos(Prop_1,\ A_1,\ C_1)}$
$\mathtt{mod\_neg(B_1,\ Prop_1)}$
$\mathtt{prop\_pos(Prop_2,\ A_2,\ C_2)}$
$\mathtt{mod\_neg(B_2,\ Prop_2)}$

Then the calculus presented in Table 1 is used to compute the effect of the modified correspondences on the relation between $\delta\mathtt{C}_1$ and $\delta\mathtt{C}_2$[4]. For reasons of clarity, some boundary cases are

Combining **prop_pos** and **mod_neg**				
	$B_1 < B_2$	$B_1 = B_2$	$B_1 > B_2$	$B_1\ ?\ B_2$
$\delta A_1 < \delta A_2$	$\delta C_1\ ?\ \delta C_2$	$\delta C_1 < \delta C_2$	$\delta C_1 < \delta C_2$	$\delta C_1\ ?\ \delta C_2$
$\delta A_1 = \delta A_2$	$\delta C_1 < \delta C_2$	$\delta C_1 = \delta C_2$	$\delta C_1 > \delta C_2$	$\delta C_1\ ?\ \delta C_2$
$\delta A_1 > \delta A_2$	$\delta C_1 > \delta C_2$	$\delta C_1 > \delta C_2$	$\delta C_1\ ?\ \delta C_2$	$\delta C_1\ ?\ \delta C_2$

Table 1: A Calculus for Processing Modified Proportionalities

omitted (for instance, if one allows modifiers to be *zero*). These can be defined in a similar way. By the same token, analogous calculi are defined for **prop_neg**'s and **mod_pos**'s.

This calculus is sufficient only for computing the influence of a single modifier on a single proportionality for C. Consequently, we have to expand our calculus to incorporate multiple modifiers and multiple proportionalities. We do this by first combining multiple modifiers for each proportionality, then computing the proportionalities one by one, and finally combining the different proportionalities.

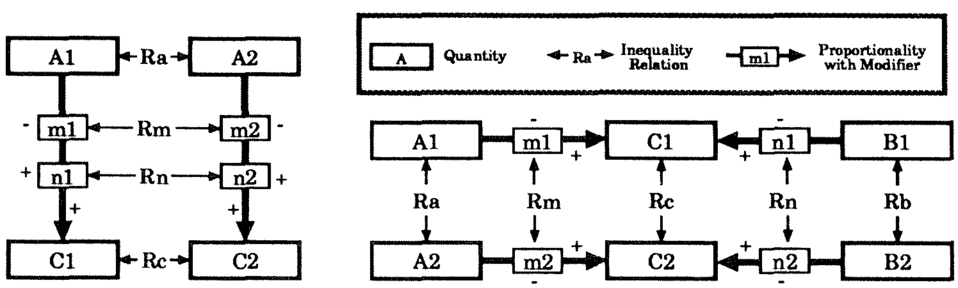

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Figure 4: Multiple Proportionalities and Modifiers

This is best explained by means of examples. In Figure 4-I, a schematic representation is given of the following relations:

$\mathtt{prop\_pos(Prop_1,\ A_1,\ C_1)}$
$\mathtt{mod\_neg(m_1,\ Prop_1)}$

[3] Because the systems are modelled by instances of the same generic model fragments, the relations $\mathtt{Prop_1}$ and $\mathtt{Prop_2}$ are abstractions of the same quantitative formula.

[4] The column for $\mathtt{B_1\ ?\ B_2}$ is added because combined modifiers can be ambiguous, as will become clear below. In subsequent tables, similar columns are omitted.

$\texttt{mod\_pos(n}_1\texttt{, Prop}_1\texttt{)}$
$\texttt{prop\_pos(Prop}_2\texttt{, A}_2\texttt{, C}_2\texttt{)}$
$\texttt{mod\_neg(m}_2\texttt{, Prop}_2\texttt{)}$
$\texttt{mod\_pos(n}_2\texttt{, Prop}_2\texttt{)}$

In addition there are inequality dependencies between the quantities (R_a, R_m, R_n, and R_c). We now want to calculate the *combined influence* of the inequality relations R_m and R_n on R_c. This is done by using a simple combination calculus, as depicted in Table 2. The right-side table applies to our example, because we want to combine a **mod_neg** with a **mod_pos**. Modifier pairs that have opposite inequality relations can be combined (*e.g.*, "$>$" and "$<$" yields "$>$", where the resulting relation is one between **mod_neg**'s), and equal modifier pairs can be omitted (*e.g.*, "$>$" and "$=$" yields "$>$"). For proportionality relations with more than

	$<$	$=$	$>$
$<$	$<$	$<$	?
$=$	$<$	$=$	$>$
$>$	?	$>$	$>$

Similar relations (*e.g.*, **mod_pos** with **mod_pos**)

	$<$	$=$	$>$
$<$	?	$>$	$>$
$=$	$<$	$=$	$>$
$>$	$<$	$<$	?

Opposite relations (*e.g.*, **mod_pos** with **mod_neg**); resulting relation is of type defined in the top row)

Table 2: Calculi for Combination

two modifiers, the calculus can be applied incrementally. The result of this addition is that all proportionalities can be represented as having exactly one modifier, because non-modified proportionalities can be seen as having one modifier which is equal in both systems. Similarly, the different proportionalities for one quantity can now be combined by adding the inequality relations between modifiers and quantities. Consider the relations in Figure 4-II, illustrating the relations

$\texttt{prop\_pos(Prop}_{A1}\texttt{, A}_1\texttt{, C}_1\texttt{)}$
$\texttt{mod\_neg(m}_1\texttt{, Prop}_{A1}\texttt{)}$
$\texttt{prop\_pos(Prop}_{B1}\texttt{, B}_1\texttt{, C}_1\texttt{)}$
$\texttt{mod\_neg(n}_1\texttt{, Prop}_{B1}\texttt{)}$
$\texttt{prop\_pos(Prop}_{A2}\texttt{, A}_2\texttt{, C}_2\texttt{)}$
$\texttt{mod\_neg(m}_2\texttt{, Prop}_{A2}\texttt{)}$
$\texttt{prop\_pos(Prop}_{B2}\texttt{, B}_2\texttt{, C}_2\texttt{)}$
$\texttt{mod\_neg(n}_2\texttt{, Prop}_{B2}\texttt{)}$

The two proportionality relations influencing C are calculated separately using the calculus from Table 1[5]. This time, we do not calculate the actual inequality between δC_1 and δC_2, but we calculate the *relative influence* of each proportionality. For example, if the left side proportionality relations in Figure 4-II would have caused R_c to change from "$=$" to "$>$" provided it was the only proportionality affecting C, then we can say that the relative influence on R_c is "$>$". When we have done this for all proportionalities affecting C, then we can add them conform the calculus in Table 2.

[5] When the subscripts for a quantity are omitted, the statement holds for both instances.

Summarising, modified proportionalities are dealt with by computing the effect of the modifier on the proportionality using the calculus in Table 1. In the case that multiple modifiers exist for one proportionality, the inequality relations between corresponding modifiers are combined by using the calculus in Table 2. If more than one proportionality affects the same quantity, the relative influences of the inequalities involved are combined by the latter calculus.

5.3 The Balance Problem Revisited

Exploiting the augmentation we proposed in the previous section, we will now show that we can predict the problematic state transition in the balance problem (Figure 2, State 2 to State 3).

In State 2, we have $H_L > H_R$, $V_L < V_R$, and $W_L < W_R$. The problem was that the transition from $H_L > H_R$ to $H_L = H_R$ was not found. With our new primitive, this can be modelled adequately. We redefine the proportionality relations between the volumes and the heights as in Section 5.1.

```
prop_pos(Prop_L, V_L, H_L), mod_neg(W_L, Prop_L)
prop_pos(Prop_R, V_R, H_R), mod_neg(W_R, Prop_R)
```

Given $H_L > H_R$, and thus $\delta V_L > \delta V_R$, we use $W_L < W_R$ to derive that $\delta H_L > \delta H_R$ (see Table 1). $\delta H_L > \delta H_R$ together with $H_L > H_R$ yields the desired transition to $H_L = H_R$.

Now suppose the problem is extended by taking into account the depths (D_L, D_R) of the containers as well. Again considering the same situation as in State 2, we can now derive that if (for example) $D_L > D_R$, the resulting combined modification, and hence the state transition, is ambiguous:

if $W_L < W_R$ **and** $D_L > D_R$ **then** $M_L \ ? \ M_R$

Here M_L, M_R are the resulting combined modifiers for both sides. Applying these modifiers on the proportionality relations between volumes and heights yields $H_L \ ? \ H_R$ (see Table 1).

If, on the other hand, $D_L = D_R$ or $D_L < D_R$, the same transition to $H_L = H_R$ is found (see the left table in Table 2):

if $W_L < W_R$ **and** $D_L = D_R$ **then** $M_L < M_R$
if $W_L < W_R$ **and** $D_L < D_R$ **then** $M_L < M_R$

Applying these combined modifiers yields

if $\delta V_L > \delta V_R$ **and** $M_L < M_R$ **then** $\delta H_L > \delta H_R$

Although the balance problems may seem to be a toy domain, the observed problem occurs in many situations. On the one hand, processes may be compared in real-life situations. Also, comparisons of predicted behaviour for different values of relevant constants can be exploited for tuning equipment. On the other hand, it is often found in teaching situations; a lot of physics problems in text books are based on the comparison of two similar processes. For example:

- "Two liquids with different heat capacities are heated; which one boils first?"
- "Two different masses slide down a hill. Compute the difference in friction."
- "The front brakes of a car are unequally worn out. In what direction will the car turn if you brake?"

By taking into account the influence of constant values, a better understanding of the process is possible.

6 Discussion and Concluding Remarks

We showed that non-changing quantities may have a significant influence on the prediction of behaviour of physical systems. When comparing similar processes, which regularly occurs in teaching situations as well as in real-life applications, current QR techniques are not capable of modelling the desired behaviour. We presented a new ontological primitive that enlarges the scope of qualitative reasoning by modelling the changes in behaviour that result from influences of non-changing quantities.

Two implementations have been discussed. They differ with respect to the explicit representation of the multiplication and the addition of modifiers. The latter is explicit with respect to its role in the prediction of causal behaviour, whereas the multiplication relation can also be used for other (teaching) purposes, for instance providing general background knowledge about the relations between the different quantities. That is, it may be useful to teach a student the multiplication relation explicitly, and not only its causal (more implicit) consequences. On the other hand, knowing which quantity is the modifier (thus by explicit representation) simplifies the realisation of causal explanations for a teaching system.

The extension proposed here is a small, but nevertheless important step. In the development of QR research, the power of qualitative reasoning has increased by gradually lowering the level of abstraction. First, only (qualitative) quantity values were used. An important improvement was the introduction of inequality reasoning, which employs the quantitative values of quantities. We follow this line one step further by introducing the modified proportionality, facilitating the exploitation of constants in the comparison of similar behaving systems. When predicting the behaviour of solitary systems these constants can be omitted, but when comparing similar systems they can influence the behaviour significantly.

In the context of our current research project on teaching qualitative reasoning, the new primitive will be employed to facilitate better explanation and cognitive diagnosis. Especially in tutoring situations, comparison of similar systems is very useful for bringing about a better understanding of the relative influences of different quantities on a system.

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