A Qualitative Images Fusion

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Abstract: Qualitative techniques usually imply some compromise between the amount of information we can deal with and the simplicity, velocity or easy understanding of the computations. In this paper we focus our attention on the enormous amount of information that any image can supply, and how the exceedingly complex problem of treating it fast can be reduced. In order to accomplish it, we introduce a qualitative image codification and implement a software tool able to tell whether two images are contradictory or not, giving us a measure of their similarity. Our goal was twofold; on one hand, we provided the raw visual data with some structure, by fixing the resolution level (granularity) and adopting an adequate codification, and, on the other hand, we built a software tool able to tell us if two of such qualitative images are able to be fused into a combined one, produce a similarity index between them and obtain the new more refined image.

1 Introduction

Day after day, qualitative techniques are spreading their application range, finding room for them in every field where the available information is too scarce for a numerical process, but, thought it can seem paradoxical, also in fields where the potential data are too many to manage. Lack of information is probably the commonest case, forcing Qualitative Reasoning to try to get the maximum knowledge from the available data, without imposing a numerical description. Methods and techniques have been developed to deal with qualitative information , and tools have been built to make an automatic reasoning process based on that data.

On the other hand, an excess of information can also disable our ability to obtain useful knowledge, for instance, a human operator overwhelmed with computer data absolutely loses his/her ability to recognize dangerous operation states. This state of things is particularly well illustrated by the case of visual data, where the enormous amount of information to be treated -probably one million pixels fifty times per second- can rapidly lead us into an exceedingly complex problem. The possibility exists, of course, of trading off some information for simplicity's sake, and returning to manageable problems. Moreover it is a good idea to reduce the detail level in order to achieve simpler and faster computations.

This will be our work in this paper. First, we will give structure to an image, by introducing a qualitative codification and establishing a determined granularity. This technique is well known in computer graphics treatment, and corresponds to scalable pixels, macro-pixels, being the resolution level defined by their sizes and adapted to every particular application we need. Our chosen codification, introduced in this work, and profiting from qualitative algebras, records only three classes of macro-pixels or cells: cells that have some object within them, cells known to be completely empty and unexplored cells. We have arbitrarily marked them with the "+", "-" and "?" signs, meaning, respectively, occupied (even partially) cells, empty cells and unexplored ones. It should be noticed that simply by choosing this codification (cells that are partially occupied considered full) we are giving priority to the presence of an object, again because of the particular application we planned it for. In any case, we get a qualitative image at a given resolution level.

In order to clarify why these choices have been made, we can think about a particular application, for instance a robotic explorer which can wander within an unknown domain and construct a map of the obstacles it finds. If we are interested only in getting the trajectories the robot can follow, it makes no sense to choose a resolution level higher than the size of the robot. This way, the minimum size of the cells is determined, while the maximum size, rather useless, would be the whole domain to explore. The qualitative map our explorer would produce would look like a grid each of whose squares is marked with the "?", "-" or "+" signs, standing for "zone unexplored", "zone I can walk into" and "zone to avoid", respectively. This application was the first we were thinking of when we design our technique, but is not the first to be completely implemented, because now we are working on a simpler quality control system, identifying rotated manufactured items.

After fixing the granularity level, the next step is working with two different qualitative images and wondering whether they can possibly reflect the same reality, or if they are contradictory, always at a fixed granularity level. Again, we can think of two robotic explorers and two different maps, and the need to test whether they represent either exactly the same scenery -maybe from a different point of view-, overlap or are completely different. Also, we can imagine that our problem is to recognize some machinery items from images obtained by static cameras, and we need to find out if two items are equal even if their images are not -rotations and translations do not alter the items but change their images-.

Of course, in any case, the answer to this question could be categorical (yes/no), but we have also introduced a similarity index that gives us a real coherence measure (remember that if the images represent an overlapping reality, they would only agree partially). This similarity index ranges from -1 to 1, its positive values corresponding to indistinguishable images and the negative values to contradictory ones. According to the qualitative codification, two images have the possibility of being equal when they are qualitatively equal, in the classical sense, zone by zone.

Nevertheless, if we try to test not the images but rather the reality they reflect, we must be aware that some transformations alter the images conserving the same reality. In this paper we have centered our work on rotations, for a number of theoretical and practical reasons, and tried to limit their *a priori* infinite number. So we must try every significant -at the chosen resolution level- transformation in one of the images, and again test the coincidence with the other one. The number of possible transformations for a given image could be enormous, but we take advantage of the limited resolution level and the geometrical simplicity of squared cells. Anyway, when we come to practical implementation we learn that every transformation from one qualitative image into another can lose some more information, and this fact has to be taken into account afterwards.

In the next sections, we are going to define how we can efficiently encode visual data, how we can determine whether they are contradictory or not and, in the last case, how to combine them to obtain a more defined one. In the first place we will try to fuse qualitatively equal images, but afterwards we will try to fuse modified views of the same reality. Then we will study the combination operator (fusion). Finally, we will determine the different qualitative rotations for a given number of cells -resolution level-, present the algorithm that implements it all and draw some conclusions. Of course, this paper is only a first step into the qualitative treatment of transformed images and we hope to fulfill their possibilities in future works.

Practical implementations are easily derived from the algorithm, since we only need a visual images acquisition able to binarily distinguish foreground and background. Then the user fixes a resolution level and the objects are approximated to this grid.

2 Qualitatively Equal Images:Fusion

A qualitative description of some information always means, at least, adopting a finite set of possible values and accepting some loss of precision. This way we increase the abstraction level but reduce the resolution (detail level).

We consider that our information is acquired in the form of a squared screen, integrated by a non relevant number of pixels. Then we reduce the detail level by defining a smaller number of squared cells, that is, partitioning X and Y axes in n discrete positions. Each of these cells will be assigned a pair of indices and will be a member of the set:

$$C = \{(i,j) / i,j \in \{1,2\ldots,n-1,n\}\}$$

We define a *qualitative image* as the map between the cells set and the set S = {+, -, ?}, so that:

$$q(i,j) = \begin{cases} ?, \text{ if the cell } (i,j) \text{ is not explored;} \\ -, \text{ if the cell } (i,j) \text{ is empty;} \\ +, \text{ otherwise.} \end{cases}$$

Notice that we want "-" to stand for a cell that is known to be empty, "+" for a partially or totally occupied cell, and "?" is assigned to every unexplored cell, that keeps the possibility of being either empty or full. Thus a qualitatively described image is a matrix, I_q, each of whose positions, $I_q(i, j)$, is mapped this way, and so a qualitative image is a member of the set of order n squared matrices whose elements belong to S, noted by $M_n(S)$.

We can see that matrices which do not contain any question mark give us the maximum description for a given image at a determined granularity level. The set of these matrices will be called $D_n(S)$.

Of course, in every particular application these matrices could be restricted in some way, meaning that not every squared order n matrix could represent valid visual qualitative data. For instance, a matrix with the form:

$$\begin{pmatrix} + & + & + & ? \\ + & - & + & ? \\ + & + & + & ? \\ ? & ? & ? & - \end{pmatrix}$$

could never have been generated by a robotic explorer which is prevented from stepping into an occupied cell, since from whichever starting point, it is impossible to step into cells (2,2) and (4,4).

Our interest now is to determine when two q-images can represent the same reality. It is obvious that this is true when they both are coincident cell by cell, that is, they have the same associated matrix. But we have also considered the "?" sign, and it implies that two non-literally equal matrices can still represent the same reality, because this sign retains the possibility of being either "+" or "-" when we acquire more information. This possibility of being equal is formalized in the definition of qualitatively equal q-images, in the following terms:

Definition 2.1: Two qualitative images, I_q and I'_q are *qualitatively equal*, denoted by $I_q \approx I'_q$, if there does not exist any cell *(i,j)* such that $I_q(i, j)$, $I'_q(i, j) \in \{+, -\}$ and $I_q(i, j) \neq I'_q(i, j)$.

This definition states that two q-images are q-equal if they are not contradictory, in the sense that there exists a cell marked with the sign "+" in one of them but with "-" in the other.

When two images turn out to be qualitatively equal, the possibility exists to combine them into a more defined image -one that possesses equal or less unexplored cells-. In order to do this we have defined the binary external operator * defined by Table 2.1. Notice that its result belongs to S'={?, -, +, !}, where the "!" stands for either incompatibility or impossibility.

*	?	–	+
?	?	–	+
–	–	–	!
+	+	!	+

Table 2.1

It is a straightforward matter to extend this definition to the matrices set, $M_n(S)$, obtaining another binary external operator, *, that will produce as a result a matrix from $M_n(S')$. Given two matrices $I_q=(a_{ij})$ and $I'_q=(b_{ij})$ from $M_n(S)$ the resultant matrix will be $I_q * I'_q = (a_{ij} * b_{ij})$.

It is also easy to see that $I_q * I'_q \in M_n(S)$ if and only if I_q and I'_q are q-equal; in other words, the operator is internal in $M_n(S)$ only when q-equal matrices are operated.

Definition 2.2 Let there be two qualitatively equal matrices I_q and $I'_q \in M_n(S)$. We define the *static fusion* of these matrices as $I_q * I'_q = (a_{ij} * b_{ij})$.

Notice that asking the q-equality of the matrices to be fused forces the result to belong to $M_n(S)$, ensuring a correct definition. On the other hand, it can be noticed that the number of question marks in the result matrix is lesser or equal than this number in any of the operated matrices, and the equality will only hold when $I_q * I'_q$ is equal to either I_q or I'_q

3 Qualitatively Equal Realities: τ-Fusion

In the preceding section we have treated the fusion between two images from a static point of view, and we have not considered what happens when both images represent the same object but are either taken from different points of view or the object has suffered some rigid transformation(s) between the acquisition of the images. In this case, the q-images will turn out not to be q-equal according to definition 2.1, but still we know that they represent the same object, as can be seen in Fig. 1:

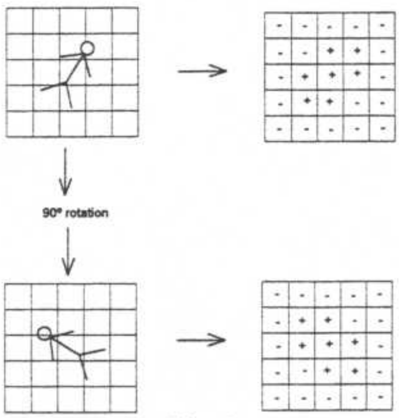


Fig. 1

On the other hand we are interested in getting the most described image possible, that is, the image holding fewer question marks, and that could be accomplished by somehow composing the data provided by two images known to represent the same reality. When this happens we must not fuse the raw images, but rather one of them with a certain transformation of the other. The kind of transformation possible depends on the particular application, but, because of their great practical interest, we have focused our present work on rigid movements, that is, transformations that preserve distances.

If we remember that the plus sign stands for a totally or partially occupied cell, it seems clear that any rigid transformation will give priority to this sign over the two others, and then to the question mark -as it holds the possibility of being a plus sign- over the minus sign.

The result is that every q-transformation implies some loss of precision, since the original image was first qualitatively encoded, but now it is numerically transformed and its result encoded again. As a consequence, when we transform one image and try to fuse it with another non-transformed one, this last image holds more precise information, and it should be preserved by introducing some asymmetry in the operation. The crucial point is that a plus sign can be incorrectly generated by the transformation, and this should not lead us to conclude a contradiction (see Fig. 2).

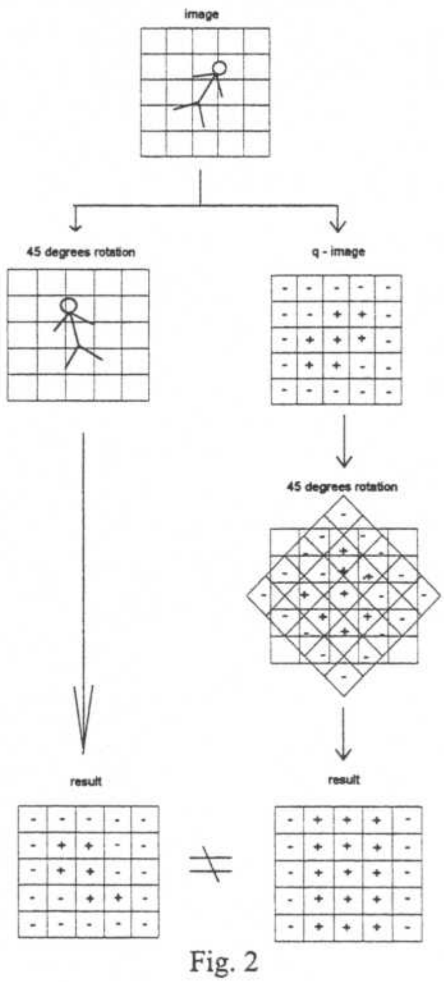


Fig. 2

And that creates the necessity of a new binary external qualitative operator, $*_d$, defined:

$$*_d : S \times S \longrightarrow S$$

$*_d$	?	−	+
?	?	−	+
−	−	−	!
+	+	−	+

Thus, when we find "+ $*_d$ - = -" it means that we give more credit to the second image, the one which had not undergone any transformation. It is worth noticing that transforming images will never lead us to an incorrect fused image, but can drastically reduce its similarity index.

Now we can extend the operator between matrices in the usual way, that is, if $I_{q_1}=(a_{ij})$ and $I'_{q_2}=(b_{ij})$:

$$*_d : M_n(S) \times M_n(S) \longrightarrow M_n(S');$$
$$I_{q1} *_d I_{q2} = (a_{ij} *_d b_{ij})$$

If we consider **T** the set of every possible mapping except Identity from $M_n(S)$ to itself, and let τ be a subset of **T**, then we can speak about:

Definition 3.1: We say that two q-images I_1 and I_2 are τ**-fusible** if there exist $t_1, t_2 \in \tau$ such that $t_1(I_1) *_d I_2 \in M_n(S)$ and $I_1 *_d t_2(I_2) \in M_n(S)$.

Physically, we say that two q-images can represent the same reality when they are τ-fusible.

The reason for needing two transformations is to ensure the operation symmetry, since no constraints are imposed on the considered subset of transformations, τ, and this definition is necessary in order to take into account the fact that two non q-equal images can still represent the same reality. Please observe that there exists the possibility that more than one transformation of an image makes it fusible with another image, and thus the set resulting from a τ-fusion between two images does not need to be a singleton. This set will be denoted by $F_d(I_1, I_2)$.

Every matrix belonging to $F_d(I_1, I_2)$ is a q-image that includes the information from the two operated q-images, possesses a lesser or equal number of question marks than any of them, but also has a possible "noise" due to the priority given to the plus sign in our transformations. This way, it is interesting to know which images from $D_n(S)$ can describe the same reality, since they are the ones that maximize our information. In this sense it holds that if $I_q, I'_q \in M_n(S)$ are such that $I_q \in F_d(I_1, I_2)$ and $I'_q \in D_n(S)$, then both q-images can represent the same reality if and only if either $I_q \approx I'_q$ or $I_q *_d I'_q \in M_n(S)$.

The kind of transformations applicable to a q-image always imply a certain movement suffered either by the point of view or the viewed object, since they both can be indistinguishably modeled. In this paper we have studied only rotations because of their practical interest, but the methodology is general enough to be applied to any kind of transformations.

4 Similarity Index

A similarity index is a measure of the coincidence between two q-images. There exists the need for such a measure in the static fusion case in order to determine the credit we can give to the resulting image, but this need is highly increased in the dynamic case because there can be many results, and we need a criterion to determine which one better meets our requirements.

This index must be designed to, on the one hand, determine if two images are contradictory or not, and, on the other, provide an idea of the degree of either contradiction or coincidence. This result must always go along with the fused image, since two completely undefined images -full of question marks- will produce a result neither contradictory nor useful, and precisely the utility of the reesult is what can be measured by the similarity index.

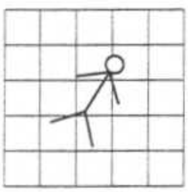
Thus, we need a index that is categorical in some sense but able to further distinction. We have chosen it to be a signed real number, whose negative values will stand for contradiction and the positive ones for coincidence. It can be normalized to range from -1 to 1, and these singular points, altogether with the 0 value are assigned to singular conditions.

We impose the similarity index to take its maximum value in the complete coincidence case -all cells explored and coincident, indistinguishable images at the given resolution level-, its minimum reflecting full contradiction -all cells explored and contradictory-, and its zero value to express neither coincidence nor contradiction -all cells unexplored. Priority is given to contradiction, and whenever it exists the similarity index takes negative values, meaning that images can never be the same. In between values are assigned by linear interpolation.

The way to achieve these goals is by defining the index as a function of the number of contradictions and coincidences, at a granularity level defined by nxn cells. At this level the similarity index between two images I_1 and I_2 is:

$$I_s(I_1, I_2) = \begin{cases} -\dfrac{contradictions}{n^2}, & \text{if } contradictions > 0; \\ \dfrac{coincidences}{n^2}, & \text{otherwise.} \end{cases}$$

Of course, when two images produce a similarity index equal to 1 it means that they are indistinguishable at a given resolution level, not that they are identical, as can be seen in Fig. 3:



-	-	-	-	-
-	-	+	+	-
-	+	+	+	-
-	+	+	-	-
-	-	-	-	-

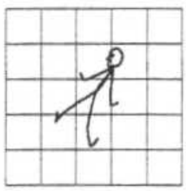


Fig. 3

5 Qualitative Rotations

Within this section, we will limit the studided rotations, g_α, where α is the rotated angle, to those made with respect to the image center, c_p. This implies that when our grid has a side with an even number of cells, this center lies on a vertex, and when this number is odd, on the middle point of a cell.

It can be seen that many real rotations can correspond to a q-rotation. If we put onto the original grid R another grid R' that has been rotated with respect to c_p, then we will not be able to distinguish the results cell by cell, no matter which angle is rotated, until a vertex from R' falls on a cell side in R. This leads us to define:

Definition 5.1. Two rotations, g_α and g_β, are *q-indistinguishable* if either $\alpha = \beta$ or for any g_γ such that $\alpha < \gamma < \beta$, the grid g_γ (R) has not got any vertex on any cell border of R;

As can be seen in its definition, this is an equivalence relation, so we can classify the real rotations into groups, in such a way that all of them belonging to the same class are q-indistinguishable for a given resolution level. This is why we call each of these classes a *q-rotation*.

It is interesting to observe that the q-description is unique for a real rotation, but usually many real rotations correspond to a same q-rotation. Nevertheless, there exist four q-rotations that are equivalence classes with cardinality equal to 1, corresponding to rotation angles $0, \pi/2$, π and $3\pi/2$. This four unitary rotations have the property of preserving the number of "+", "-" and "?" cells for every q-image, so they are called *neat rotations*. Any non-unitary rotation applied to a q-image can increase the number of either "+" or "?" signs, introducing, respectively, noise or more indetermination in the q-image. They are called *dirty rotations*.

To put it more simply, two rotations are q-indistinguishable when, for any real image, the q-description obtained after applying either one or the other is the same.

Now we study for the different values of N, the order of the q-image matrix, which q-rotations can be distinguished. We look for a partition of the interval $(0, 2\pi]$ corresponding to the different q-rotations. We will center on the first quadrant, since the series of angles will repeat itself starting from every neat rotation. Thus we find a sequence of angles between 0 and $\pi/2$ such that every interval $(\alpha_i, \alpha_{i+1}]$ represents a different q-rotation.

For each grid, we only consider the vertices belonging to the most external level of cells. This forces us to take into account every grid R_n contained in R with lower order. It means that, when N is odd, n is odd and ranges from 3 to N, and when N is even, n is also even and ranges from 2 to N.

Given a grid R_n, we have to find all the rotation angles that make any of these rotated vertices lay on a straight line belonging to the grid.

In order to find these angles, it is useful to take a reference in polar coordinates, which express the vertices as:

$$\left(\sqrt{k^2+i^2}\right)_{\beta_i} \text{ and } \left(\sqrt{k^2+i^2}\right)_{\beta_i'}$$

$$\text{with } \beta_i = \operatorname{arctg}\frac{i}{k} \text{ and } \beta_i' = \operatorname{arctg}\frac{k}{i}$$

where, if n is odd we have $k = n$ and i is a natural odd number varying from 1 to k. If n is odd we have $k = n/2$ and i is a natural (not necessarily odd or even) number varying from 0 to k.

It should be noticed that when rotating a vertex, m_β within the first quadrant, if it falls onto the vertical line $x = z$, then the vertex with the same module and complementary argument, $m_{(\pi/2)-\beta}$, falls onto the horizontal line $y = z$. In other words, it can be proved that, if g_α (m_β) meets the line $x = z$, then $g_\alpha(m_{(\pi/2)-\beta})$ meets the line $y = z$. Because of this property, we can focus our study on the intersections between vertices belonging to the first quadrant and vertical straight lines, in order to find the border angles between two classes (q-rotations).

That leads us to the necessity of solving the system for x and α:

$$\left.\begin{aligned} &x = z \\ &x = \sqrt{k^2+i^2}\cdot\cos(\beta_i+\alpha) \end{aligned}\right\}$$

where i and k vary as we have specified before, and z is an integer number ranging: when n is odd, from the ceiling of $-n\sqrt{2}$ to the floor of $n\sqrt{2}$, taking only the odd values; and when n is even, from the ceiling of $-n\sqrt{2}/2$ to the floor of $n\sqrt{2}/2$, taking both the odd and the even values.

For each set of values the solutions are found:

$$\alpha = \arccos\left(\frac{z}{\sqrt{k^2+i^2}}\right) - \operatorname{arctg}\frac{i}{k};$$

$$\text{and } \alpha' = \arccos\left(\frac{z}{\sqrt{k^2+i^2}}\right) - \operatorname{arctg}\frac{k}{i}.$$

These values, after being ordered, will give us the partition of the interval $(0, \pi/2]$ associated to the different q-rotations (equivalence classes).

6 Algorithm

Our software tool accepts qualitative images -images assumed to have been partitioned in cells and qualitatively encoded by another program-, performs the direct fusion and gives the similarity index. Whenever the user indicates that this index is too low, and automatically when it is negative, our program tries every significantly different rotation and computes the τ-fusion again producing a similarity index and a combined image.

The first thing the user must specify is the granularity level, that is, the desired number of cells, and then he/she is asked to introduce the qualitative visual data of two images. Because of the possibility of varying the resolution level, every data structure in the program is dynamic.

The fusion algorithm is simple, and goes no further than a cell by cell implementation of the binary operator that refines information, also recording each coincidence and contradiction to compute the similarity index. The difficulty appears in the τ-fusion operator, when we need to test every possible rotation; theory gives us an easy way to compute the border angles, but we still have not found the relation between each angle partition and the resulting image. Meanwhile, our algorithm performs a numeric rotation by selecting an intermediate angle within each partition and geometrically finding the projection of our rotated grid onto a static one. This last point, finding out how many and which cells of the static grid are "stained" by each rotated cell, is the computationally most expensive.

A first solution would have been implementing a classical graphic algorithm testing within which static cells every border of the rotated cells falls, but this process is obviously slow and complex. The chosen method takes advantage of the convexity and geometrical simplicity of a squared form, when we are working within a Cartesian space, and it is based on the fact that the center and corners of a square cover nearly all the possible "stained" static cells, and the special case when a cell is only stained by a segment of the rotated square can be solved with a complementary view. This last approach finds which vertices of the static grid lie within the rotated squared cell (that is why we call it the complementary point of view); there are just three possible cases: either no vertices found, which means that every cell has already been marked, or only a vertex is found, meaning that the four neighboring cells will result stained, or two vertices are found and six cells will be marked, in vertical or horizontal direction. This quest for vertices is the most difficult point in the whole algorithm, and maybe it is worth a second look:

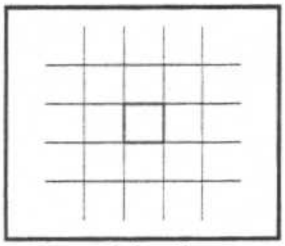


Fig. 4

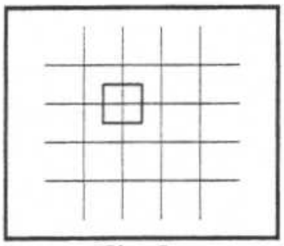


Fig. 5

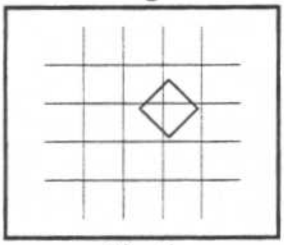


Fig. 6

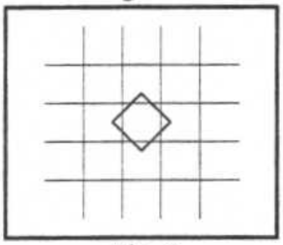


Fig. 7

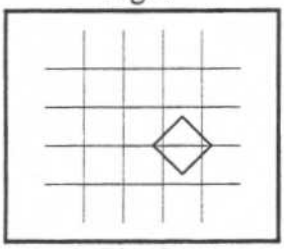


Fig. 8

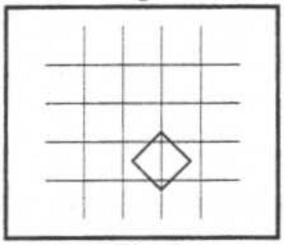


Fig. 9

To determine how many vertices lie under a particular transformed cell, we first take the coordinates of its four corners and convert them into rows and columns, by taking the integer part of those coordinates divided by the cells length From them we select the maximum distance in each direction and distinguish the four cases reflected in: Fig 4) both distances are 0, no vertex can be contained; Fig. 5) and Fig. 6) both distances are 1, one vertex must be contained, and four cells are to be marked; Fig. 7) both distances are equal to 2, again no vertex of the static grid is found within the rotated cell; Fig. 8) and Fig. 9) distance equal to 2 in one direction and to 1 in the other, two vertices, six cells to be marked, interpolating in the direction of maximum distance. If we have stored the values of coordinates that produced the maximum distance, their combination gives us the cells to mark.

The outline of the basic algorithm can be given this way:

Input of the granularity level.
Input of two q-images.
Direct fusion of both images, producing a new image and a similarity index.
Determination of the least informative image.
For *every significant angle at this resolution*
For *every "-" cell*
Rotate the cell
Mark the "stained" cells on a static grid
End for
For *every "?" cell*
Rotate the cell
Mark the "stained" cells on a static grid
End for
For *every "+" cell*
Rotate the cell
Mark the "stained" cells on a static grid
End for
Dynamic fusion of both images, producing a new image and a similarity index.
End for

The result supplied by our algorithm is twofold: the collection of all possible fused images for every rotation angle and their associated similarity indices. This last output really expresses the confidence we can put on the fused image. If we think of our explorer robot, we can work with the resulting map whenever the similarity index is positive, but our confidence increases as the index does, until we are sure, when it reaches its maximum value of 1, that the two maps are indistinguishable at the fixed resolution level. When we try to recognize only whether two objects have equal shapes, the fused image holds no interest and only the similarity index will guide us.

7 Conclusions. On Going Research

In this work we have shown the interest of applying qualitative techniques to reduce the complexity of visual data treatment. This complexity comes from the overflowing low-level information they contain, and our first step was to study a higher level, more abstract, description of the data.

Then we wondered about when two images, encoded this way, could be considered equal, and when they could be considered at least not contradictory. Both cases were studied and an index of similarity was designed.

We also admitted incomplete qualitative images, that is, images containing unexplored cells, and we tried to combine two of them into a new one. When operating with two complete images, the result only tells us if they correspond or not to the same reality, but if they contain unexplored cells their combination results in a more complete image. This way we aggregate information, and this process can be repeated time after time.

If we only use the test for two images to represent the same reality, a direct application is item classification, implemented by trying to fuse the unknown object with the ideal standard form. Many industrial processes need to recognize items, for instance their products, in order to control their quality, and also many administrative processes need to recognize when one item has the appropriate form. Of course at present there exist other techniques can solve this problem, but they need to know in advance the object to be recognized, and extract its invariant spatial properties. Our tool can work automatically by means only of the visual real-time acquired data.
On the other hand, we can start with incomplete images (unexplored cells) and get a more refined combination of them. Of course, the more obvious application in this field is the fusion of two incomplete maps, obtained from incomplete and independent explorations of some scenery. We can think of two robotic explorers that return to their base where our tool tries to combine the information they provide. This is the application we mainly think of, and that is why we give priority to the presence of some object within a cell, because an empty cell means that our explorer is able to cross it freely, and a full cell means that it is not.

There are many fields open to future research. Among them we can remark: the interest in trying a more complex qualitative set -orders of magnitude- to model more complex decision processes, where some risks can be taken; the possibility of a complete description of a qualitative geometry, taking into account any transformation; and, to conclude this brief list, the further study of the similarity index, how it should be formalized, its evolution as information is aggregated and how to correctly use it in real problems ...

Turning our look to practical applications, we are at the present working in a quality control system. Images of industrial manufactured items are taken, and they have to be compared with an "ideal image", in order to be either accepted of rejected. Of course, the possibility exists to get the image of a rotated item, and classical techniques fail to cope with this situation. Our tool simply gets the image and binarizes it according to a grid that fixes the granularity level; then it tries every possible rotation and answers the question of a possible identity between the inspected an the ideal item. Whenever this answer is no (negative similarity index), the possibility of being equal does not exist, but when the answer is yes (positive index), it just means "maybe", and in the case of a returned value near one, it means "the two images are equal at this resolution level". Still if the user is reluctant to accept this conclusion, he/she can increase the resolution level (number of cells) and test again the possible identity.

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