Design Platform for Planar Mechanisms based on a Qualitative Kinematics

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Abstract: Our long term aim is to address the open problem of combined structural/dimensional synthesis for planar mechanisms. For that purpose, we have developed a design platform for linkages with lower pairs which is based on a qualitative kinematics. Our qualitative kinematics is based on the notion of *Instantaneous Centers of Velocity*, on considerations of curvature and on a modular decomposition of the mechanism in *Assur groups*. A qualitative kinematic simulation directly generates qualitative trajectories in the form of a list of circular arcs and line segments. A qualitative simulation amounts to a propagation of circular arcs along the mechanism from the motor-crank to an effector point. A general understanding of the mechanism, independent of any particular dimensions, is extracted. This is essential to tackle synthesis problems. Moreover, we show that a dimensional optimization of the mechanism based on our qualitative kinematics successfully competes, for the *path generation problem*, with an optimization based on a conventional simulation.

1 Introduction

Mechanical linkages with lower pairs are planar mechanisms of fixed topology, consisting of rigid bodies constantly connected with joints of revolute type (allowing a rotation) or prismatic type (allowing a translation). An example is given in Fig. 1. Linkages with lower pairs represent a large part of the existing mechanisms in industry. Our subject is to develop an efficient automatic or semi-automatic method for the synthesis of these linkages. Such a method should find one or several optimal mechanisms submitted to functional requirements. These requirements are conventionally:

- the desired kinematics of one or several bodies,
- no collision between the mechanism and its environment,
- the minimization of the efforts in joints when the two previous criteria are fulfilled.

For the moment, we have limited our research to:

- linkages of one-degree-of-freedom,

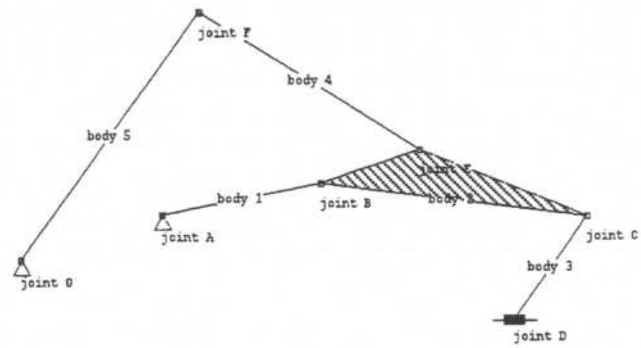


Figure 1: Example of a linkage, composed of a *slider-crank mechanism* (bodies 1, 2 and 3) and a *dyad* (bodies 4 and 5). Joints A and G are revolute joints (rotation) connected to the fixed frame. Joint D is a prismatic joint connected to the fixed frame. The other joints are revolute joints connecting two bodies. There is a motor at joint A, so body 1 is a crank.

- a requirement of desired kinematics in the form of the *desired trajectory* of a point, termed *effector point*, on a body of the mechanism, making abstraction of the orientation of this body. This problem is called the *path generation problem*.

A synthesis process consists of choosing the type of mechanism (*type synthesis*), e.g., slider-crank or four-bar mechanism, and the optimal dimensions for this mechanism (*dimensional synthesis*). The dimensional synthesis problem has been widely studied for two decades and more [13; 25], and some satisfactory techniques have been implemented. However, the combined type/dimension synthesis problem remains an open problem for two major reasons. On one hand, it often consists in testing a huge number of simple equivalent mechanisms in terms of a unique criterion: the number of degrees of freedom [12; 24; 26]. On the other hand, the criteria which preside over the pruning of such a combinatory are completely questionable. In fact, the choice of the type of mechanism is very little constrained by the requirements and, for a given type of mechanism, there is no classification of the trajectories in terms of the dimensions. This reveals that at present we have very poor and unstructured knowledge about the interactions between functions, structures and dimensions for the mechanisms.

It is for this reason that we developed a platform for the design of linkages which is based on a qualitative kinematics. The idea of this qualitative kinematics is to consciously accept a small loss in numerical accuracy of trajectories, in order to handle more global and qualitative information and to have a deeper understanding of mechanisms. This is made by directly generating qualitative trajectories in the form of lists of circular arcs and line segments, instead of generating lists of points like in a conventional simulation.

Firstly, we show that a dimensional optimization (synthesis) process based on (looping over) such a qualitative kinematic simulation is faster than an optimization process based on a conventional simulation. For brevity, we will term these two dimensional optimization processes: *qualitative optimization* and *exact* (or conventional) *optimization*. Secondly, and this is the real challenge, we show how to construct a qualitative understanding of a mechanism as an engineer could do, in order to justify the qualitative shape of the trajectory from particular dimensions. Only the work of Shrobe [27] deals with this purpose but in a less explicit manner. The *qualitative explanations* that our system generates use two mechanisms of causal deductions: a modular decomposition of the mechanism and a causal graph of *Instantaneous Center of Velocity*[1] calculations. Thirdly, we hope that, in the long term, we shall be able to invert the explanation process. Then, we would be able, in a combined type/dimension synthesis process, to intelligently index some mechanism types which are serious candidates and to directly propose value intervals for the dimensions.

Section 2 presents the two functionalities of the design platform: generation of the qualitative explanations and qualitative optimization. We evoke, in section 3, the modular decomposition of a mechanism and its advantages. In section 4, we present our qualitative kinematics. Section 5 shows how our qualitative kinematics can be applied to a dimensional optimization. The most important section is section 6 where we show how to construct qualitative explanations for a mechanism. Finally, in section 7 we compare our work with others on qualitative kinematics.

2 The two functionalities of the design platform

2.1 Qualitative explanations and qualitative dimensional optimization

Both functionalities of the design platform (generation of the qualitative explanations and qualitative optimization) require as a first step an approximation of an exact trajectory (in the form of a list of points) into a qualitative trajectory (in the form of a list of arcs and segments). In both cases, this resulting qualitative trajectory is used to drive a qualitative kinematic simulation.

[1] Henceforward, they are termed ICs.

In the case of the generation of qualitative explanations, the exact trajectory is generated by a primary conventional kinematic simulation for specific dimensions of the mechanism which must be explained. This is the trajectory of the effector point which intervenes in the specification, i.e., in the function of the mechanism, so that we can say that the sought function drives the explanations. The process of the generation of the qualitative explanations is summed up in Fig. 2.

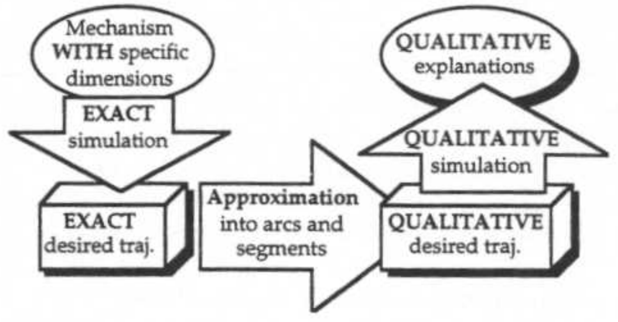


Figure 2: Generating qualitative explanations of the behaviour of a mechanism.

In the case of qualitative optimization (which is a loop over a qualitative kinematic simulation), the exact trajectory is specified by the designer himself because this is the desired trajectory to fulfill. After a qualitative dimensional optimization, optimal dimensions for the mechanism are proposed: this is the *qualitative optimum* (see Fig. 3). In order to confirm the validity of our approach (in spite of the two consecutive approximations: first approximation into arcs and segments and qualitative simulation), we will have to compare the *qualitative optimum* with the *exact optimum* directly obtained by a conventional optimization.

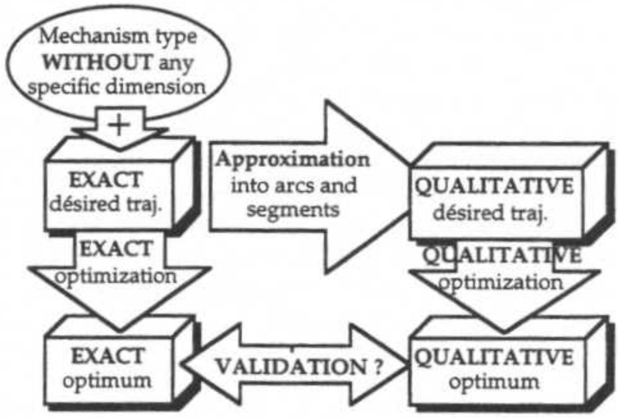


Figure 3: Qualitative dimensional optimization. The method remains to be validated by comparison between the qualitative and the exact optima.

2.2 Approximation of an exact trajectory into arcs and segments

This approximation takes as input argument a maximum admissible error. This error is the maximum distance

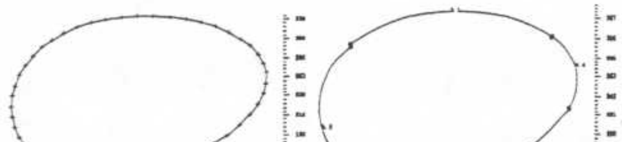


Figure 4: Approximation of an exact trajectory (left) defined by 48 points into a qualitative trajectory (right) defined by 4 circular arcs.

between a point of the exact trajectory and the qualitative trajectory. A minimization of the total number of arcs and segments is carried out with a least square algorithm. For the same total number of arcs and segments, the maximum number of segments is preferred because they have more significant qualitative properties. Finally, the third criterion is to prefer the approximation which minimizes the error. The algorithm does not respect the continuity between the arcs (and segments) because it is not necessary. An example of approximation is given in Fig. 4.

3 Modular decomposition of linkages

Modular decomposition of linkages is an efficient approach for linkage analysis and a way to better understand a mechanism [4; 10; 17]. It consists in dividing the mechanism into elementary structural groups, called *Assur groups*. An *Assur group* is obtained from a kinematic chain of zero mobility by suppressing one or more links, at the condition that there is no simpler group inside. Theoretically, the number of *Assur groups* is not limited, but practically only the simplest are used.

An interesting extension of the theory of *Assur groups* consists in a systemic view of the modular approach. Pelecudi [20; 21] extended the notion of *Assur group* to the concept of a *multipole group* as an elementary sub-system possessing input poles (joints), motion and effort transfer functions, internal poles and output poles (see Fig. 5). Input and output poles are considered as potential joints, allowing connection with other groups. We have implemented in our platform an algorithm for the automatic modular decomposition of linkages. This decomposition is unique (see Fig. 6).

A kinematic simulation consists in looping over a configuration determination corresponding to a particular crank angle. During a configuration resolution, each group,

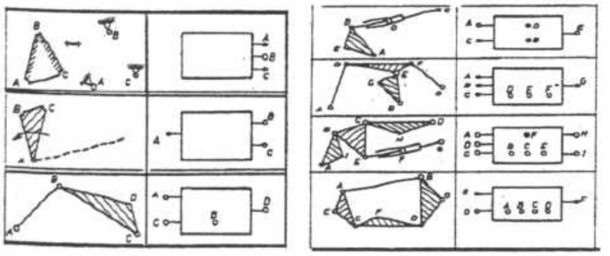


Figure 5: Examples of the simplest *Assur groups* and the corresponding systemic icons.

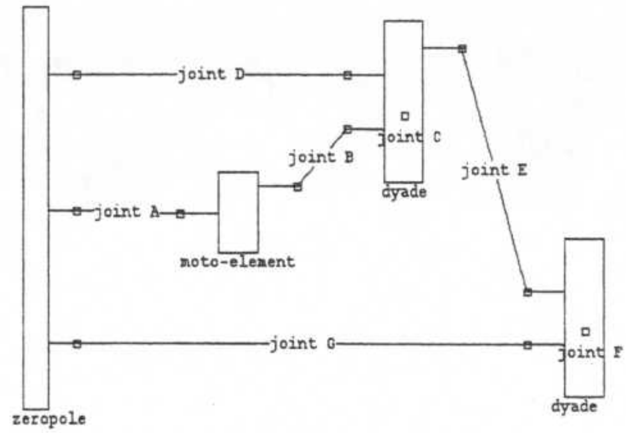


Figure 6: A systemic representation of the linkage of Fig. 1 by a modular decomposition into *multipole groups*.

considered as a sub-system, receives from the previous group information about the position, velocity and acceleration of its input joints. After an internal calculation according to the transfer function of the group, the kinematic features of the output poles are determined and transmitted to the next group.

Thus, a configuration resolution can be viewed as a motion propagation from the fixed frame to the terminal group. In normal situations, the terminal group contains the effector point in order for each group to contribute to the position and velocity of the effector point. Otherwise, the superfluous groups must certainly have another function in the mechanism and the motion of their internal poles will be interesting for future qualitative explanations. This is the case of the linkage in Fig. 1 with an effector point in E; point F (see Fig. 6) is detected as an *interesting point* for explanations.

The transfer function is known a priori for each type of *Assur group*. For most of the simpler groups, the position, velocity and acceleration of the output poles are expressed in an explicit manner from the kinematic features of the input poles. Such a modular decomposition is a systematic approach to reduce iterative calculations for any type of linkages. It can be considered as a symbolic compilation of the mechanism, depending only on its type and not on its particular dimensions. It allows a kinematic simulation method which is more efficient than computing and solving equations for the whole mechanism at once. It can help kinematic (and also kinetostatic) analysis, but also synthesis (for more details, see [30]).

In our platform, the modular decomposition will be used for a resolution in positions (not in velocities) inside a qualitative kinematic simulation.

4 The qualitative kinematic simulation

The qualitative kinematic simulation is the basis of our work. In very few iterations (corresponding to the number of arcs and segments), it generates a "rough" simulation in terms of numerical accuracy but its interest is double:

- to obtain a good elementary calculus time inside a dimensional optimization process,
- to build qualitative explanations of mechanism behaviour.

This qualitative simulation is based on the notions of *Instantaneous Centers of Velocity* (ICs) and some notions of curvature in plane kinematics.

4.1 An approach based on the Instantaneous Centers (ICs)

During the movement of a mechanism, it can be considered that the relative movement between any couple of bodies (say i and j) is locally[2] equivalent to a rotation of center IC_{ij}. This is the *Instantaneous Center of Velocity* between the bodies i and j. Of course, if bodies i and j are connected by a revolute joint, IC_{ij} is constantly the center of the joint, fixed relatively to body i and body j. Otherwise, IC_{ij} changes over time. In like manner, a prismatic joint between bodies i and j, is kinematically equivalent to a revolute joint whose the relative IC is at infinity in a direction D_{ij} orthogonal to the direction of translation. Traditionally, the ICs are used in graphical methods to calculate a specific absolute velocity (relative to the fixed frame 0) or a relative velocity between two bodies, starting from the known velocity of the motor-crank as in the one-degree-of-freedom mechanism of Fig. 1. A remarkable property of these ICs is given by the *theorem of three centers*:

The relative ICs of any three bodies: IC_{ij}, IC_{jk} and IC_{ik} lie on the same straight line.

Now, let us consider a particular position of the motor-crank for which the rotational velocity is known. Let us denote M a point on the motor-crank, hence its velocity vector is supposed to be known. The IC between the fixed frame (0) and the motor-crank (M, too), denoted IC_{0M}, is constantly the center of the fixed revolute joint. Let us denote T, standing for trajectory, the effector point and, by extension, the body where it is attached. The knowledge, for a particular crank position, of the positions of the effector point and ICs: IC_{MT} and IC_{0T}, leads to the calculus of the velocity vector of the effector point relatively to the fixed frame, as explained in Fig. 7. These two interesting ICs are not *a priori* known.

It is worth knowing that when IC_{MT} and IC_{0M} are geometrically equal, the effector point has a zero velocity (it locally stands still).

[2] "Locally" means during an infinitesimal period of time.

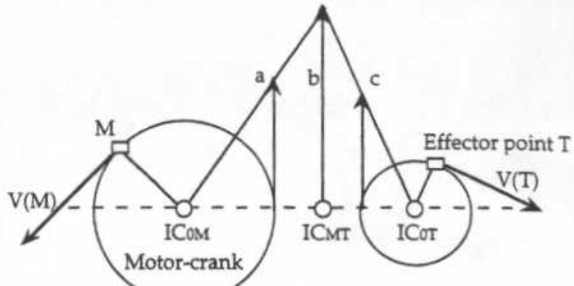
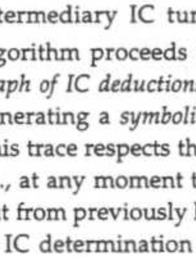


Figure 7: Calculation of the effector point velocity vector. Graphically, the method is simple: the velocity vector of M and vector a have the same norm, the velocity vector of the effector point T and vector c too. Vectors. a, b and c are orthogonal to the straight line of the ICs.

We have developed an algorithm which provides the method to determine the unknown ICs from the known ICs by using the *theorem of three centers* and with a minimum of intermediary ICs. According to the *theorem of three centers*, two known ICs: IC_{ij},and IC_{jk} define the straight line where lies IC_{ik}, termed ΔIC_{ik}. In like manner, a known IC: IC_{jk} and a direction D_{ij} coming from a prismatic joint, define a straight line ΔIC_{ik}. The elementary step of the algorithm is the determination of a new IC IC_{ik} by intersection of two known straight lines ΔIC_{ik}.

Let us take the mechanism of Fig. 1 and suppose that the positions of the joints are known. Hence, six ICs are already known from a total of fifteen (C_6^2): IC_{01} is joint A, IC_{12} is joint B, IC_{23} is joint C, IC_{24} is joint E, IC_{45} is joint F, IC_{05} is joint G. The effector point is E and an *interesting point* is F (see also §6). These points are both fixed to body 4. We saw that we were to determine IC_{MT} and IC_{0T}, i.e. in our case: IC_{14} and IC_{04}. For this case, only one intermediary IC turns out to be necessary: IC_{02}. The algorithm proceeds in two steps: building first a *causal graph of IC deductions* (see Fig. 8.a) and, from this graph, generating a *symbolic trace of IC deductions* (see Fig. 8.b). This trace respects the sequentiality of the IC calculations, i.e., at any moment the calculation of a new IC is carried out from previously known or calculated ICs. The method of IC determination can be graphically displayed to the



(a)

(b)

$$IC_{02} = (IC_{01}, IC_{12}) \cap (D_{03}, IC_{23})$$
$$IC_{04} = (IC_{02}, IC_{24}) \cap (IC_{05}, IC_{45})$$
$$IC_{14} = (IC_{01}, IC_{04}) \cap (IC_{12}, IC_{24})$$

Figure 8: An *IC causal graph* (a) and the corresponding symbolic trace (b) which shows the order of IC calculations.

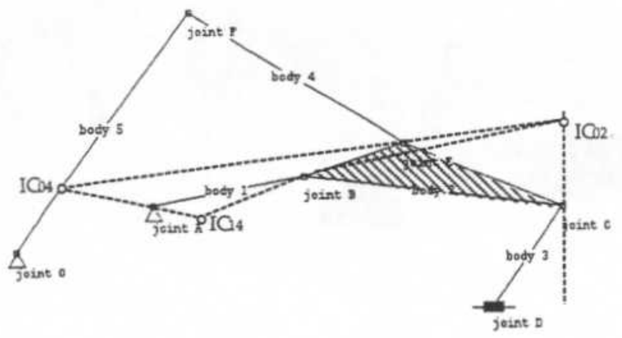


Figure 9: Method of determination of the sought ICs for the mechanism of Fig. 1.

designer (see Fig. 9). This algorithm of IC determination is applied once for a mechanism type and a given effector point; it is independent of any specific dimensions. The symbolic trace of deductions is used for automatically generating the coordinate equations of the sought ICs and the velocity vector equations of points E and F (as explained in Fig. 7) in a symbolic form. In this manner, velocity vectors are calculated without any iteration and by the call to an optimized procedure. Like for the modular decomposition, it can be said that the mechanism is "compilated". The fact that there is no iteration for the determination of velocity in a general case of mechanism is a good result from the point of view of speed. Even the modular approach of *Assur groups* does not escape to iterative calculations of the velocity for some (rare) *Assur groups*.

4.2 Curvature of qualitative arcs and centrodes

Our goal is not to calculate the velocity vector of the effector point from the rotational velocity of the motor-crank, but it is to directly obtain a circular arc (or a segment) centered on a position of the effector point and corresponding to a finite rotation of the crank. The center and the radius of curvature of the trajectory of the effector point is thus needed at this particular position. But the center of curvature of this arc is not the *Instantaneous Center of Velocity* IC_{0T}, as we could expect. Indeed, IC_{0T} is locally the center of relative rotation, but the following instant IC_{0T} has moved. We can have a better understanding in considering the loci of the ICs in the fixed frame (0), namely the *fixed centrode*, and the loci of the ICs in the frame of body T, namely the *moving centrode*. By rolling the moving centrode attached to body T about the fixed centrode, the motion of body T is faithfully represented (see Fig. 10). It is clear that the IC: IC_{0T}, the center of curvature and the effector point are aligned.

Problems of curvature in planar motion have been widely studied by mathematicians during the eighteenth and nineteenth centuries [31]. The radius of curvature of the arc can be calculated by the *Euler-Savary equation* [13; 25]. Although the *Euler-Savary equation* can be solved by a graphical method, this is not a purely qualitative theory like the deductions of ICs by the use of the *theorem of three centers*. Presently, we do not think that explanations for understanding the mechanism could be easily extracted. Instead, we use a numerical correction which is presented further. But, for the purpose of explanation generation, some concepts are worth studying further:

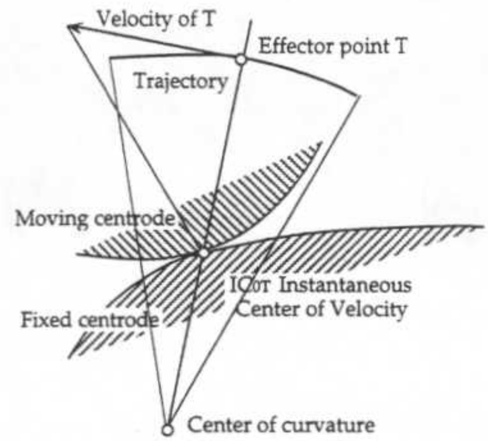


Figure 10: Moving and fixed centrodes.

- the *inflection circle* which is the location of effector points on body T whose curves have a segment,
- the *cubic of stationary curvature* which is the location of effector points whose curves really contain a circular arc.

4.3 Propagation of circular arcs along the mechanism

Now, let us examine the problem of the correspondence between the limiting points of the arcs and segments of the desired qualitative trajectory of T, and the positions of the motor-crank M. Let us take the example of Fig. 11 where the approximation of the exact trajectory gives one segment and three arcs.

The four corresponding positions of the crank, which have to be determined, are parameterized by the angle θ_0 corresponding to the initial position 0 and three angles θ_{0i} (i varying from 1 to 3), relatively to this position.

In the case of the generation of qualitative explanations, the exact desired trajectory is provided by a primary conventional simulation. Therefore, it is possible to represent the curvilinear abscissa of the points of the exact desired trajectory function of the relative angle of the motor-crank. An interpolation between these points gives the function $sT = f(\theta_{relative})$ (see Fig. 12). The angles θ_{0i} are straightforwardly determined from the curvilinear abscissas sT_i of the limiting points of the arcs and segments.

In the case of a dimensional optimization, there exist two types of *path generation problem* according to the specification:

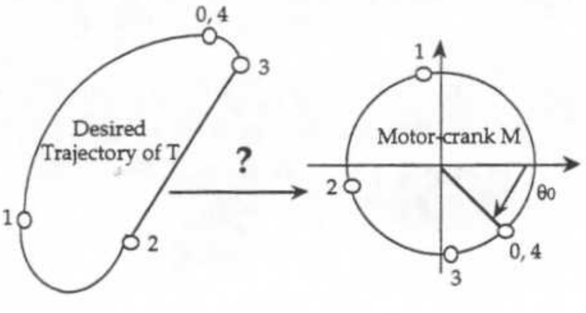


Figure 11: What is the correspondence between the limiting points of the arcs and segments of the desired qualitative trajectory of T and the positions of the motor-crank ?

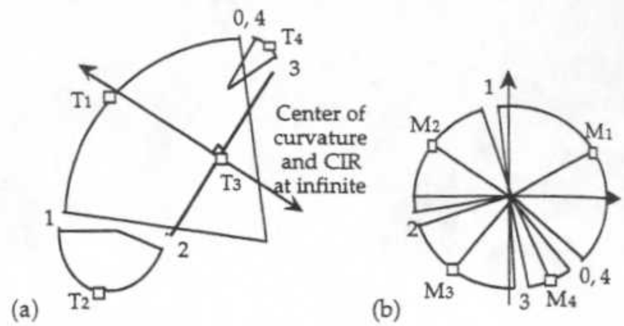


Figure 13: (a) Arcs and segments of the qualitative trajectory of the effector point T. (b) Arcs of the qualitative trajectory of point M attached to the motor-crank.

- The effector point must move along the desired trajectory in accordance with prescribed temporal positions of the crank. It amounts to impose the curve $sT = f(\theta_{relative})$, i.e. angles θ_{0i} corresponding to sT_i. If this curve is the straight line $((0,0),(2\pi, sT_{max}))$, it means that the curvilinear velocity of the effector point and the angular velocity of the motor-crank are proportional. The designer can also define areas of relatively high acceleration or deceleration for the effector point.
- The effector point must move along the desired trajectory whatever the corresponding positions of the crank may be. Thus, angles θ_0 and θ_{0i} become design variables which are to be valued during the optimization process. This represents the general case.

Now, let us consider the mid-points M_i of the circular arcs of the trajectory of M (see Fig. 13.b) and the mid-points T_i of the circular arcs and line segments of the qualitative desired trajectory (see Fig. 13.a).

In saying that points T_i temporally correspond to points M_i, we commit a piecewise linear approximation of the curve $sT = f(\theta_{relative})$ (compare Fig. 14 and Fig. 12). In fact, the effector is considered to move with a constant velocity along a circular arc (for a constant rotational velocity of the motor-crank). Again, this acceptable approximation confirms that we construct a qualitative kinematics.

From now, we will consider, at the inverse, a qualitative kinematic simulation where the circular arcs of the current trajectory of the effector point must be calculated from the circular arcs of the motor-crank. For each position of the motor-crank in M_i (the mid-point of a circular arc), a resolution of positions is carried out, using the graph of modular decomposition; the positions of the effector point T_i and the positions of the joints are then determined. The velocity vector of the effector point $V(T_i)$ is determined by the specific procedure generated from the *symbolic trace of IC deductions*. This vector gives the orientation and the length of the arc centered on T_i. Indeed, benefiting from the fact that the ratio of the curvilinear velocities are constant on an arc, the curvilinear total lengths of the two corresponding arcs respect the same ratio. Thus, we directly consider that the velocity values are equal to the curvilinear length values. At present, the determination of the radius of curvature is solved by a second position resolution of the mechanism very close from the mid-point of the arc T_i. It can be seen in Fig. 10 that this second position is sufficient because we already know the straight line (IC_{0T}, T_i) where the center of curvature lies.

The deduction of the velocities from the motor-crank to the effector point through multiple ICs can be viewed as a propagation of a velocity vector through the mechanism. Therefore, since a velocity vector and a circular arc are somewhat equivalent (except for the radius of curvature),

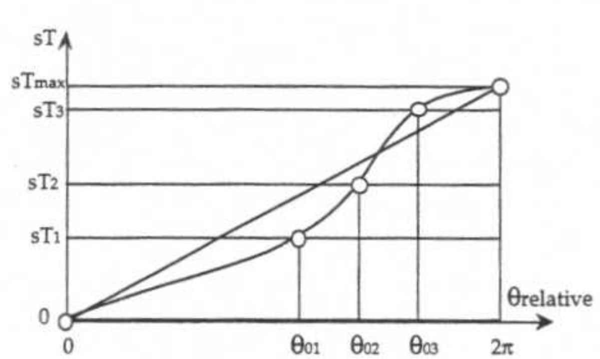


Figure 12: The knowledge of the curve $sT = f(\theta_{relative})$ leads to the determination of angles θ_{0i}.

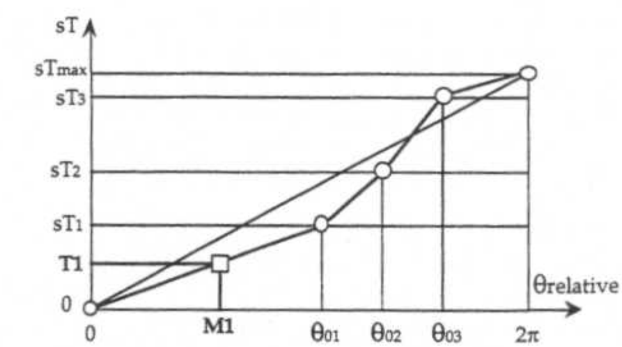


Figure 14: The piecewise linear approximation which is made because of the use of qualitative arcs and segments (to be compared with the original curve in Fig. 12).

the image of the propagation of circular arcs from the motor-crank to the effector point can be adopted.

5 Dimensional optimization based on qualitative kinematics

In this section, our concern is to dimensionally optimize a given mechanism for the *path generation problem*, i.e. to find the best dimensions in order for an effector point to move along a desired trajectory.

Several types of methods are currently used: graphical techniques [13; 25], direct analytical methods [2] and optimization techniques (non-linear programming) [1; 3; 4; 11; 18; 24; 28; 32]. A major drawback in using analytical methods is that the number of prescribable points[3] of the desired trajectory is limited by the type of mechanism. Therefore, for a large class of practical design problems, we need optimization methods, which represent a general design tool. However, this is a hard problem especially because of its high non-linearity.

After having specified an exact desired trajectory, the designer approximates it into a qualitative desired trajectory (let us take the example of Fig. 4). Next, the designer chooses a mechanism type (the one of Fig. 1) and specifies the effector point (E) and the design variables which can vary in the optimization process. By default, the coordinates of revolute joints and the angles of prismatic joints, corresponding to the initial configuration of the mechanism, are design variables. The coordinates of the effector point are not design variables because the effector point is constrained to be at the position 0 of the desired trajectory. The bar lengths are deduced from these variables. Moreover, angles θ_0 and θ_{0i} of the motor-crank, corresponding to the qualitative desired trajectory, must be added, when no temporal constraint is specified on the motion of the effector point. In the example of Fig. 1, it can be deduced from the modular decomposition of Fig. 6 that the *dyad* group composed by bodies 4 and 5 does not intervene in the motion of the effector point E. The general design vector is then:

$$(x_A, y_A, x_B, y_B, x_C, y_C, \alpha_D, \theta_0, \theta_{01}, \theta_{02}, \theta_{03})$$

Here, an important advantage of the qualitative approach is that the number of variables of angles (θ_0 and θ_{0i}) is equal to the number of qualitative arcs (and segments). For a conventional approach, this number would have been equal to the number of points of the desired trajectory. In the example of Fig. 4.b and 4.a, these numbers are respectively 4 and 48.

In fact, for a conventional approach with numerous precision points, another technique is used to avoid this design variable explosion. The aim is to make abstraction of any temporal correspondence between the two trajectories, only the final curves are then compared. In [11], for example, the calculated trajectory is interpolated by a spline for each simulation and is compared to the spline interpolation of the desired trajectory. The penalty function between the two trajectories is the sum of squares of two sets of points equally spaced along each spline. But, these interpolations are extremely costly. Therefore, in our qualitative approach, we prefer to have, for the sake of efficiency, a few additional design variables for the crank angles and to perform a one-to-one comparison between the two lists of arcs and segments. The cost due to the distance between the desired and the simulated trajectories takes into account the coordinates of the limiting points A and C of the arcs and segments and their mid-points B, according to the formula:

$$F_{dist} = \frac{w_1}{Nb.arcs} \sum_{arcs} \left[\begin{array}{l} \frac{1}{2}\left((x_{Ad} - x_{As})^2 + (y_{Ad} - y_{As})^2 \right) + \\ \left((x_{Bd} - x_{Bs})^2 + (y_{Bd} - y_{Bs})^2 \right) + \\ \frac{1}{2}\left((x_{Cd} - x_{Cs})^2 + (y_{Cd} - y_{Cs})^2 \right) \end{array} \right]$$

The factor $\frac{1}{2}$ for the limiting points comes from the fact that these points are counted twice, in two neighbour arcs or segments. Even if they are not identical because of the non-continuity, they correspond logically to the same point on the desired trajectory, so they must be counted only once. w_1 is a weight coefficient.

The optimization algorithm must penalize the mechanisms which encounter blocking positions during a simulation. For that purpose, we propose a penalty term which is proportional to the square of the difference between the curvilinear length of the desired trajectory (L_d) and the curvilinear length of the simulated trajectory (L_s). When, during a simulation, a blockage is encountered, the simulation is rerun backwards in order to obtain the maximum value for L_s. So, we have a penalty term: $F_{block} = w_2(L_d - L_s)^2$. Finally, the term F_{constr} is introduced in order to take into account user defined constraints such as: the linkage must never cross a delimiting box, one bar length must be lower than a value, and so on. So, the global penalty function is:

$$F = F_{dist} + F_{block} + F_{constr}$$

We adopted an optimization algorithm based on the gradient method because of the non-accessibility to partial derivatives, the non-linearity and inequality constraints. Anyway, the choice of the optimization algorithm turns out to be almost independent from the type of kinematic simulation (qualitative or not). For the example of Fig. 1 (11 design variables), the use of a qualitative simulation instead of a conventional simulation speeds up the optimization, practically with a factor of two, which corresponds the speed ratio between two simulations of different type. Indeed, instead of dealing with a description

[3] We find also in the literature: *precision points*.

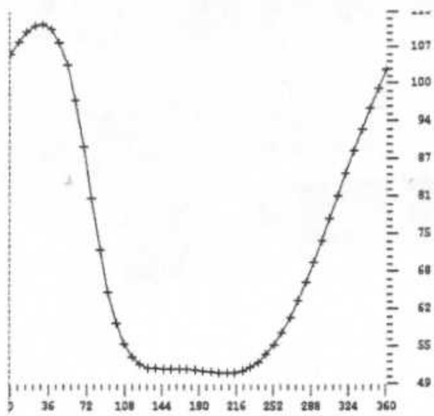


Figure 15: The curve of the angle of body 5 (see Fig. 1) as a function of the angle of the motor-crank exhibits a period of constant angle named a *dwell*.

of the desired trajectory by a collection of points, qualitative simulation deals with a much smaller number of arcs and segments while at the same time almost faithfully reproducing the exact curve when the mechanism reaches ideal dimensions. Finally, we have confirmed, on several examples, the validity of our model of qualitative optimization in finding a qualitative optimum very close to the exact optimum.

6 Understanding linkages

The title of this section is deliberately the same as an article by Shrobe [27]. Below, we will compare both approaches of understanding linkages. Presently, let us take again the mechanism of Fig. 1. The effector point is joint E and its exact trajectory is given in Fig. 4.a. This exact trajectory is approximated into four circular arcs with a high degree of accuracy (see Fig. 4.b). The mechanism exhibits the same phenomenon as the mechanism of Shrobe's article, namely a *dwell*. Indeed, observing the curve of the angle of body 5 as a function of the crank angle (see Fig.

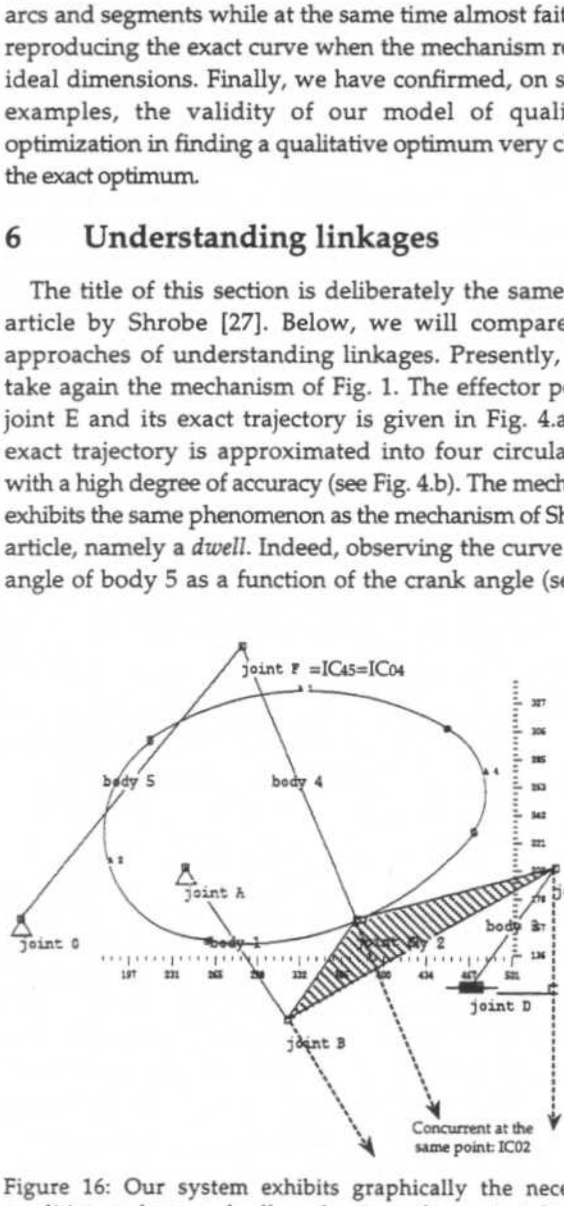


Figure 16: Our system exhibits graphically the necessary condition to have a *dwell*-mechanism: three straight lines attached to bodies must be concurrent.

15), we notice that body 5 stands still during a rotation of the motor-crank of about 140°; this is the *dwell* phenomenon.

This is due to the fact that when the effector point is moving along arc A3 (see Fig. 16), the length of body 4 is such that joint F is constantly at the center of curvature of arc A3. Our system does not only describes this phenomenon but it also explains why it occurs for the particular dimensions of the mechanism. This is an essential difference with the work by Shrobe.

Joint F was, for us, an *interesting point*. When we try to determine the velocity of joint F, for the position corresponding of the mid-point of arc A3: T_3, we fall on the particular case (see Section 4.1) where IC_{45} and IC_{04} are geometrically equal, i.e. the distance is lower than a fixed admissible distance. Thus, we have proved symbolically that body 5 stands still locally for the position corresponding to T_3. This is a necessary condition for body 5 to stand still for a period of time (*dwell*). The second necessary condition is that the center of curvature is also geometrically identical IC_{45} (joint F), which is immediately shown. Concerning the first condition, our system is capable of generating many more relevant explanations, in explaining why, in this particular position, the dimensions of the mechanism caused IC_{45} and IC_{04} to be geometrically equal. According to the causal graph of ICs, we know that IC_{04} is obtained by:

$$IC_{02} = (IC_{01}, IC_{12}) \cap (D_{03}, IC_{23})$$
$$IC_{04} = (IC_{02}, IC_{24}) \cap (IC_{05}, IC_{45})$$

With IC_{45} and IC_{04} identical, the system infers, after some symbolic handling, that the three straight lines (IC_{01}, IC_{12}), (IC_{45}, IC_{24}) and (D_{03}, IC_{23}) are concurrent at the same point IC_{02}. This result is presented graphically to the designer which understands that: " *the bar of body 1, the bar of body 4 and a vertical bar passing through joint C are necessarily concurrent* " (see Fig. 16).

From particular *dimensions* of a mechanism of a given *type*, we have extracted a general understanding of this mechanism and we have given dimensional conditions to respect in order to perform the *dwell function*. This last feature is not implemented by Shrobe. However, this generalization of the knowledge linking type, dimensions and functions is an essential condition to efficiently tackle the problem of combined type/dimension synthesis.

7 Related works

Our work addresses both the domain of linkage synthesis and the domain of qualitative kinematics. We already stated (in the introduction) that the recent works on a combined type/dimension synthesis of linkages [24; 28] were not satisfactory because of the poorness of the knowledge about the interactions between mechanical functions, mechanism types and mechanism dimensions. This is why we developed a qualitative kinematics to extract general explanations on the mechanism behaviour from particular dimensions.

The two primary major approaches in qualitative kinematics were those of Joskowicz & al [14; 15; 16; 19; 22; 23] and Faltings & al [5; 6; 7; 8; 9]. The two approaches are very similar in principle:

- they are dedicated to mechanisms of changing topologies, with contacts of complex shapes,
- the configuration space of a mechanism is defined by combining both quantitative and symbolic information,
- a qualitative kinematic simulation consists in forecasting the state transitions, i.e. changes in the topology (a contact is established or broken).

On one hand, Joskowicz & al consider three-dimensional mechanisms with fixed axes; on the other hand, Faltings & al consider two-dimensional (or planar) mechanisms of two degrees of freedom like a pawl and ratchet [6; 7], or clocks [9].

In [7], Faltings and Sun began to explore the relationships between the part dimensions and the kinematic functions. Given a mechanism and some initial dimensions, they propose a technique to redesign the system with new dimensions in order to meet functional requirements.

Subramanian and Wang [29] defined a synthesis tool of three-dimensional, fixed topology, single-degree of freedom mechanisms. This tool proposes to assemble elementary mechanisms of fixed axes in order to meet global kinematic requirements.

But these works are dedicated to distinct classes of mechanisms from that of our work: the linkages (planar mechanisms of fixed topology) with lower pairs. Consequently, they can not really be compared with ours.

Only the work of Shrobe [27] which we mentioned above deals with linkages. Shrobe does not develop a qualitative kinematics. He just considers the result of a conventional kinematic simulation, i.e. the trajectories of *interesting points*, that he approximates into circular arcs and line segments. The *interesting points* are, principally, the input/output points of *building blocks*, sorts of modular groups as ours. He detects some similarities between the features of the qualitative curves (e.g. same interval of time for two arcs). His concern is to construct an understanding of a mechanism in conjecturing causal relationships between these qualitative features. Because his model does not include deep knowledge, like our qualitative kinematics based on the ICs and curvature considerations, his explanations will remain hypothetical and incomplete. Moreover, we are convinced that conjectures can only generate judicious explanations in very rare cases, for specific dimensions of the mechanism. Anyway, we claim that there is no conjecture to be made, because all the relationships between an arc of the desired trajectory, the corresponding arcs of the motor-crank and other interesting points, and the particular dimensions of the mechanism, are given from the ICs chaining and considerations of curvature. The consequences in our understanding of a mechanism is that Shrobe explains that there is a *dwell* (see section 6) because joint E revolves around joint F on a circular arc of appropriate radius, whereas we explain why this circular arc exists (because three straight lines related to the mechanism are concurrent) and because we relate dimensions to kinematic functions.

8 Conclusion

We have presented a new qualitative kinematics for mechanical linkages, which is based on a deep knowledge of the planar kinematics: considerations of *Instantaneous Centers of Velocity* and of centers of curvature. A qualitative kinematic simulation amounts to propagate a circular arc of trajectory from the motor-crank to an effector point, thanks to an *IC causal graph*. It permits to extract a general understanding of the mechanism from a specific simulation. This understanding relates for a mechanism type the dimensions to the kinematic functions (e.g., a *dwell*). This knowledge relating type (structure), dimensions and functions is essential to tackle the combined type/dimension synthesis, which is still an open problem for linkages. We already showed that the use of a qualitative kinematic simulation in a dimensional optimization process (for the *path generation problem*) successfully competes with a conventional approach.

Moreover, we know that our qualitative kinematics theory is extensible to:

- several effector points and corresponding desired trajectories,
- rotational motors not related to the fixed frame,
- translational motors,
- several degrees of freedom,
- gear pairs.

Our future work will consist in exploiting the qualitative explanations of linkages into a general synthesis system.

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