A Qualitative Model of Physical Fields

Monika Lundell
Artificial Intelligence Laboratory
Computer Science Department
Swiss Federal Institute of Technology
IN–Ecublens, 1015 Lausanne, Switzerland
lundell@lia.di.epfl.ch

Abstract

A qualitative model of the spatio-temporal behaviour of distributed parameter systems based on physical fields is presented. Field-based models differ from the object-based models normally used in qualitative physics by treating parameters as continuous entities instead of as attributes of discrete objects. This is especially suitable for natural physical systems, e.g. in ecology. The model is divided into a static and a dynamic part. The static model describes the distribution of each parameter as a qualitative physical field. Composite fields are constructed from intersection models of pairs of fields. The dynamic model describes processes acting on the fields, and qualitative relationships between parameters. Spatio-temporal behaviour is modelled by interacting temporal processes, influencing single points in space, and spatial processes that gradually spread temporal processes over space. We give an example of a qualitative model of a natural physical system and discuss the ambiguities that arise during simulation.

Introduction

Research in qualitative physics has so far mostly focused on lumped parameter models of man-made physical systems, e.g. refrigerators and electrical circuits. A lumped model describes the temporal but not the spatial variation of the parameters within a system.

This paper focuses on distributed parameter models, describing temporal as well as spatial variation. These models are especially appropriate for natural physical systems that have so far received little attention in qualitative physics, e.g. the atmosphere, the ocean and many other environmental and ecological systems.

Scientists often think of distributed parameter systems in terms of physical fields. A physical field describes the spatial distribution of the values of a parameter. Its properties are functions of space coordinates and of time. This view contrasts with the object-based ontology commonly used in qualitative physics, where models are constructed around a set of interacting objects described by several physical parameters. In a distributed parameter system like the atmosphere, there is usually no obvious object structure to associate the parameters with. Instead, with a field-based view, the parameters of the system are seen as evolving patterns determined by the current spatial configuration of values. The patterns are combined spatially as needed to obtain a more complete view of the system, which evolves due to physical processes acting in regions where patterns intersect.

In this paper, we present a qualitative model of the spatio-temporal behaviour of distributed parameter systems based on physical fields. The model is divided into a static and a dynamic part. The static model describes each parameter as a qualitative physical field by dividing space into contiguous regions according to the quantity space of the parameter. The pattern of a field is described by its regions' boundaries and contiguities. Composite fields are constructed from the region boundaries' coincidence and traversal properties within other fields. The dynamic model describes processes acting on the fields and qualitative relationships between the parameters. Processes are triggered by spatial features in individual or composite fields. Spatio-temporal behaviour is modelled by interacting temporal and spatial processes. Temporal processes directly influence single points in space. Spatial processes have an indirect influence by gradually spreading temporal processes over space.

The purpose of the presented model is to support the qualitative methods used by scientists. In some sciences, e.g. ecology, qualitative methods are a necessity, since many processes lack numerical models. One practical example is landscaping, e.g. envisioning the evolution of a planned garden in order to avoid undesirable events (flooding, spreading of weeds, pests, etc). This involves a number of distributed parameters (land elevation, soil quality, seed distribution, etc), whose interaction is seldom described by precise numerical models. In many cases, exact coordinates are also missing. In other sciences, qualitative methods are used as a complement to existing numerical models. One example is weather forecasting, which is carried out in two phases. The first phase, called the objective analysis, is entirely quantitative. Finite-element simulations of partial differential equations are run on huge amounts of observed data, resulting in a map of predicted data. The second phase, called the subjective analysis, is qualitative. The meteorologist analyzes both the observed and predicted data for each parameter by drawing patterns, e.g. isobars and rain regions, on the maps. This time-consuming analysis is carried out without computer support and builds up an "inner weather picture" (Perby 1988), i.e. an understanding of the current atmospheric processes that enables the meteorologist to produce a final forecast. The working methods of scientists can be characterized as qualitative, model-based and diagrammatic, thus touching on three related fields in artificial intelligence.

In the following, after a brief survey of related work, we present the static and dynamic models and a diagrammatic formalism for visualization of qualitative physical fields. We give an example of a qualitative model of a natural physical system and discuss the ambiguities that arise during simulation. We conclude by outlining further work and discussing the main contributions with respect to other research fields.

Related work

In qualitative physics, this research is related to the early work of FROB (Forbus 1983), in the sense that a qualitative physical field resembles a place vocabulary, i.e. a set of contiguous regions where some important property is constant. However, place vocabularies have not previously been used to model individual and composite parameters of natural physical systems. Instead, most qualitative models of natural physical systems are lumped and adopt the object-based ontology. The process-based ontology and the syntax used in our model have been inspired by Qualitative Process Theory (Forbus 1984).

In spatial information theory, there is extensive work on qualitative spatial reasoning. Most approaches describe relations between points and are not applicable to continuous fields. The topological properties of extended regions have been studied by e.g. (Cui, Cohn, & Randell 1992) and (Egenhofer & Al-Taha 1992). However, these approaches focus on pairs of regions and do not explicitly represent the properties and physics of continuous fields.

Geographical information systems provide methods for storage and analysis of large amounts of spatial data. Although these systems are mainly quantitative, they address many issues relevant to this research. In particular, the representation of individual parameter fields and their subsequent combination corresponds to the traditional cartographic technique of map overlay, where transparent maps of different themes are physically combined in order to produce a complete map. Map overlays can be manipulated using the map algebra of (Tomlin 1991), which, however, is not relevant to our purposes since it requires a predefined discretization of space into a grid.

Static model: structure

The static model describes the structure of a distributed parameter system as a set of qualitative physical fields. It consists of a distribution model for each individual field and an intersection model for each pair of fields that are to be combined in a composite field.

Distribution model: individual fields

Each individual parameter is described as a qualitative physical field by a distribution model. A qualitative physical field represents a double discretization:

First, the value domain of the parameter is discretized into a quantity space, i.e. a finite set of symbols called landmark values, representing qualitatively interesting events in the system, e.g. the critical values. A qualitative value either corresponds to a landmark value or to an interval between two landmark values. Representations of quantity spaces have been discussed in e.g. (Forbus 1984) and (Kuipers 1994). They are totally or partially ordered sets, whose values can only be compared for equality or order.

Second, the space of the physical field is discretized into a pattern of contiguous, non-overlapping regions corresponding to the parameter's qualitative values. The regions are maximal in the sense that no region is adjacent to a region with the same value. We represent the continuous pattern of a qualitative physical field by the boundaries and contiguities of its regions.

The boundary number of a region indicates the number of topological holes. Each region has as many internal boundaries as holes, plus one external boundary. The boundaries, in their turn, represent the contiguities of the region, i.e. its adjacencies to other regions. Each boundary is divided into a number of faces, each indicating an adjacency to another region. For two-dimensional regions, with one-dimensional boundaries, the faces can be ordered in a sequence. In three dimensions, the boundaries are themselves two-dimensional regions with boundaries whose faces can be ordered.

Scientists often use diagrammatic methods to reason about distributed parameter systems. Consequently, we have developed a diagrammatic formalism for visualization of qualitative physical fields. The diagrams are deliberately abstract in order to convey only the qualitative properties represented by the model, i.e. the presence of distinct regions, continuity, boundaries and contiguities. This avoids the problematic issue in diagrammatic reasoning that pictorial representations may be interpreted as containing more information than intended (Wang, Lee, & Zeevat 1995).

Figure 1 shows the distribution models of two different fields. The qualitative information in the left-hand pattern is conveyed by the right-hand diagram, where the regions are represented as circles and circle sectors. Internal boundaries are represented by nesting the circles and contiguities by shared perimeters. The unusual shape of the first diagram reflects the inner regions' contiguity with the outside world.

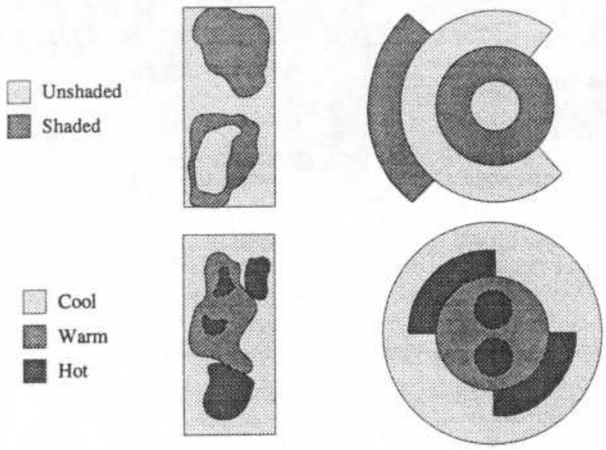

Figure 1: Diagrams of distribution models.

Intersection model: composite fields

A composite field is the spatial combination of two or more physical fields. It is constructed from intersection models describing the coincidence and traversal properties of the boundaries in each pair of fields to be combined.

For each boundary, the behaviour of its faces within the other field is described. Each face is divided into an ordered sequence of face segments representing either coincidence, i.e. a shared path with a face segment in the other field, or traversal, which means the face segment cuts through a region in the other field.

The intersection model is visualized diagrammatically as indicated in figure 2, corresponding to the physical overlay of the two patterns in figure 1. The face segments are indicated by small white squares in the diagrams. The mapping between the fields is visualized as a graph. Each face segment is connected by an edge to either a face segment (coincidence) or a region (traversal) in the other field. The figure shows a partial mapping. For clarity, the face segments in the pattern have been labelled and indicated by arrows.

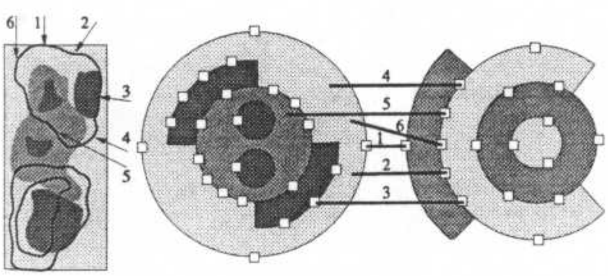

Figure 2: Diagram of intersection model.

Dynamic model: behaviour

The dynamic model describes the behaviour of a distributed parameter system in terms of processes acting on the fields, and qualitative functional relationships between the parameters.

A coherent framework is obtained by letting functional relationships and processes have spatial extent and representing them as physical fields. The regions of applicability are represented by the intersection model of a parameter field and a process or relationship field.

Functional relationships between parameters are described with the vocabulary of Qualitative Process Theory (QPT) (Forbus 1984). We write *(qprop+ parameter1 parameter2)* to indicate the existence of a function that determines the value of *parameter1* and is increasing monotonic in its dependence on *parameter2*. A decreasing monotonic relationship is indicated by *qprop−*.

We follow the ontology of QPT and use processes as the sole mechanism of change. A process is an entity that acts over time to change a parameter. The concept has so far mostly been used in lumped models describing only the temporal progress of the processes. In a distributed model, spatial progress must also be considered. We model spatio-temporal behaviour as an interaction between temporal and spatial processes.

In the following, we discuss the properties of temporal and spatial processes by developing a model of a natural physical system: heat flow in a partially shaded meadow. The system consists of three distributed parameters: irradiaton, shade and temperature. Irradiation indicates the parts of the meadow that could be irradiated by the sun. The irradiation can be considered constant for meadows of normal size, but would have a varying distribution in a model of a larger area, e.g. an entire continent. The shade parameter distinguishes between shaded and unshaded regions, e.g. due to passing clouds or obstructing trees. The dynamic behaviour of the system is modelled by two processes: 1. The temperature will rise in all irradiated regions, but less in shaded regions, 2. A varying temperature distribution will lead to a horizontal flow transporting heat from warmer to cooler regions.

Temporal processes

A temporal process is similar to a QPT process. It selects regions from individual or composite fields and imposes direct influences on their values. A temporal process is represented as a field and acts on each individual point in the selected regions. Since a region is not a fixed object, but can be decomposed with respect to other fields, parts of the region may be influenced by other processes. The process fields are composed in order to determine the net change to the values in the influenced fields and to update the region structures.

We describe temporal processes with a syntax similar to QPT's but adapted to our model as follows:

- The *regions* referred to by the process are specified by pattern templates indicating the following: 1. The name of the region, 2. The individual or composite field to which it belongs, 3. Whether to retrieve an atomic region, which is the default, or a union of contiguous regions, 4. Conditions on values and spatial features. Value conditions compare parameter values, while spatial conditions indicate constraints on boundaries and contiguities. The conditions can refer to other regions by their names. A parameter value is referred to by combining the name of the field and the name of the region.
- The *region-conditions* indicate additional conditions on values and spatial features that could not be expressed earlier. The regions and region-conditions correspond to the individuals, preconditions and quantity-conditions of QPT.
- Names of local *variables*, to be used in the process, are defined.
- The *relations* indicate functional relationships valid during the lifetime of the process. The variables and relations correspond to QPT relations.
- Direct *influences* are specified with QPT syntax. *(I+ parameter variable)* indicates a monotonic increasing influence of the variable on the parameter. *I–* indicates a monotonic decreasing influence.
- The *stop-conditions* indicate conditions on values and spatial features that stop the process.

The following example models the warming of each atomic region in the composite field of irradiation, shade and temperature. The heating-rate is inversely proportional to the amount of shade. The process stops when the irradiation is reduced to zero.

```
temporal-process solar-warming
   :regions          (r :fields      (irradiation shade temp)
                        :atomic      T
                        :conditions  (> (irradiation r) zero)
   :variables        heating-rate
   :relations        (qprop- heating-rate (shade r))
   :influences       (I+ (temp r) heating-rate)
   :stop-conditions  (<= (irradiation r) zero)
```

Spatial processes

Spatio-temporal behaviour is modelled by interacting spatial and temporal processes.

Spatial processes are different from temporal processes in that they do not act in a single point but gradually spread influences over space, starting from a boundary between two regions. A spatial process is represented as a field with expanding applicability regions, called expansion regions. The faces of the expansion regions correspond to fronts that move at a certain rate. The path of a spatial process can be guided by defining functional relationships between the rates of the fronts and the values of the regions they move through.

A spatial process can change other fields only indirectly by spreading temporal processes. Each expansion region is associated with a temporal process defined as a local variable within the spatial process. These embedded temporal processes do not themselves select a region to act on, but are applied to the points encountered by the fronts of the expansion region. Once applied, a temporal process is activated and decoupled from the spatial process. It obeys its own stop-conditions, which, however, can refer to the local variables of the spatial process.

A spatial process is defined similarly to a temporal process. The main difference lies in the influences. Since parameters are not directly influenced by spatial processes, I+ and I– are not used. Instead, the expansion regions are defined as follows:

```
(E expansion-region from-region to-region
      rate influence stop-conditions)
```

Expansion-region names a local variable for the expansion region. *From-region* and *to-region* define from which boundary and in which direction the region starts expanding at the specified *rate*. *Influence* is the name of an embedded temporal process. The *stop-conditions* for this particular expansion region are indicated. The spatial process can also have global stop-conditions, in analogy with a temporal process, indicating conditions that will stop all expansion regions.

The following example models the second process in our example, horizontal heat flow, which only concerns the temperature field. A spatial process is triggered by adjacent regions of different temperature, *r1* and *r2*. Two local variables, *heating-rate* and *expansion-rate*, are directly proportional to the temperature difference. Two embedded temporal processes are defined, *tp1* and *tp2*, that respectively increase and decrease the temperature at the specified *heating-rate* until the two expansion regions, *ep1* and *ep2*, have equal temperature. The expansion regions are spread into *r1* and *r2* respectively with the specified *expansion-rate*, starting from their common boundary, each applying a temporal process to each passed point. The expansion is defined to stop only when the other region is no longer expanding.

```
spatial-process heat-flow
   :regions    (r1 :fields        temp)
               (r2 :fields        temp
                   :conditions    (adjacent? r1 r2)
                                  (> (temp r1) (temp r2)))
   :variables  heating-rate
               expansion-rate
               (diff (- (temp r1) (temp r2))
               (tp1 (temporal-process heat-flow
                      :influences (I+ temp heating-rate)
                      :stop-conditions  (= (temp ep1)
                                           (temp ep2))))
               (tp2 (temporal-process heat-flow
                      :influences (I- temp heating-rate)
                      :stop-conditions  (= (temp ep1)
                                           (temp ep2))))
```

```
:relations  (qprop+ expansion-rate diff)
            (qprop+ heating-rate diff)
:influences (E ep1 r1 r2 expansion-rate tp1
                (not (expanding? ep2)))
            (E ep2 r2 r1 expansion-rate tp2
                (not (expanding? ep1)))
```

This example demonstrates a tricky issue with dynamic fields: the identity of a region. The temporal processes must refer to the expansion regions instead of *r1* and *r2*, since the latter cease to exist as distinct regions when the temperature starts changing. The local variable *diff* that is computed from the values of these regions must thus be considered as constant during the lifetime of the process.

This example gives a flavour of the qualitative aspects of natural physical systems that can be modelled. It can be extended with fields describing e.g. the distribution of different kinds of seeds, water availability, soil conditions, etc. Compositions of these parameters indicate varying living conditions, and can be used to model different ecological systems.

Qualitative simulation: ambiguity

The purpose of a qualitative simulation is to describe the evolution of a system as a sequence of qualitatively interesting states. In lumped models, a new state is generated each time a parameter reaches a significant landmark value, called a limit point in QPT.

We use the same technique, but additionally consider spatial limit points that are reached when the structure of the system changes. The change can either concern a distribution model or an intersection model. One example is when a region reaches the same value as one of its neighbours due to an influencing temporal process. Since all regions must be maximal, the two regions will be merged into one, thus changing the distribution model. Another spatial limit point is when an expansion region crosses a boundary in another field, which entails a change to the intersection model of the process field and the parameter field.

Since qualitative models use incomplete information, ambiguities can arise when the next limit point is to be determined. Our model inherits the ambiguities of lumped qualitative models, which can be divided into value ambiguities, e.g. deciding whether the difference of two qualitative values is less or greater than a third value, and temporal ambiguities, e.g. deciding which of two changing parameters will next reach a limit point. Distributed qualitative models additionally have spatial ambiguities, which in our case arise when determining in which order spatial limit points will be encountered by an expansion region.

In lumped qualitative models, a tree of behaviours can be generated when there is a known number of alternatives for each ambiguity. This is not the case for distributed qualitative models lacking shape information. Figure 3 shows an example of two fields whose regions have the same qualitative structure but different shapes. The shaded region is a single expansion region, in a separate spatial process field, that gradually spreads from region A into region B. At the instant indicated in the figure, the expansion region's boundary traverses region C twice in the first field, but only once in the second. The intersection model of the spatial process field and the parameter field thus cannot be unambiguously established within the framework of the qualitatative model, nor is there a known number of alternatives since region C could be of arbitrary shape.

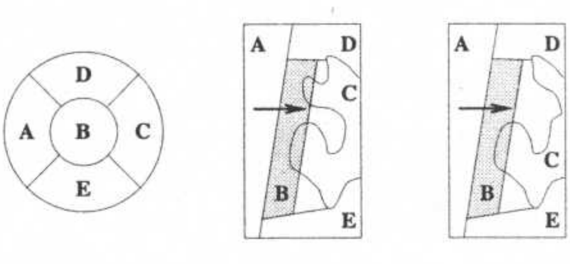

Figure 3: Fields with identical qualitative structure. The grey region is a superimposed single expansion region spreading in the indicated direction.

The existence of spatial ambiguities means that our approach does not provide a general solution to the poverty conjecture of (Forbus, Nielsen, & Faltings 1987) stating that qualitative spatial reasoning requires a metric diagram conveying shape information. However, the poverty conjecture originated in reasoning about objects in small-scale space, e.g. gearwheels, where the coordinates of the metric diagram can be obtained. We argue that qualitative spatial reasoning based on topological and ordinal information is useful for a different kind of situation, where the initial metric data is sparse or incomplete, e.g. in the form of scattered observation points. In these situations, both quantitative and qualitative simulations have to rely on assumptions and simplifications.

In the case of scientists analyzing their data, we hypothesize that intractable ambiguities, like shape, are simplified to a known number of alternatives at a cognitive level, which makes it possible to use the envisioning techniques of qualitative reasoning. We believe this is done by assuming a non-complex spatial configuration, given the known qualitative constraints, as well as a non-complex spatial evolution of the system. Note that this does not mean assuming convex or regular regions, since a continuous field, unless it is a grid, must necessarily contain concave and irregular regions.

Based on this hypothesis, the qualitative simulation algorithm generates an envisionment as a sequence of diagrams differing in as few spatial features as possible. Figure 4 shows a non-complex behaviour of the initial situation in figure 3, generated from a few simple complexity-reducing heuristics. The shaded regions indicate the intersection of the field with a single expansion region as it spreads gradually from region A into region B. In the first diagram, only B is partially covered. The least complex transition to the next state is assuming that the immediate neighbours of B are reached, i.e. D and E. In the next state, B is completely covered and C has been reached. The final least complex transition is to the state where regions B, C, D, and E are entirely covered by the expansion region. The rules governing the simulation algorithm are described in detail in (Lundell 1995).

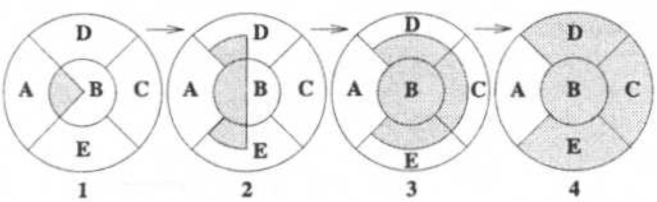

Figure 4: Qualitative simulation as a sequence of diagrams describing one possible non-complex evolution.

Conclusion

The main contributions of the research described in this paper, with respect to related research fields, can be summarized as follows:

- Qualitative physics: We introduce the concept of a qualitative physical field, describing a physical system in terms of parameters instead of objects. A technique for modelling spatio-temporal behaviour and a language for spatial processes are presented.
- Spatial information theory: We do not limit ourselves to pairs of regions, but describe the qualitative properties of continuous fields containing many regions. Spatial features are not only described topologically, but also with ordinal information suitable for qualitative analysis.
- Geographic information systems: We present a qualitative alternative to the quantitative techniques for representation and simulation of spatial data used in current systems. Qualitative methods are advantageous in situations with incomplete spatial data that cannot be satisfactorily represented in a quantitative system.

Distributed parameter systems have several additional features that can be exploited in qualitative reasoning. We are currently working on a number of related issues:

- Extending the qualitative physical fields with regions describing not only point-wise parameters but also amounts, averages and totals. This will also entail extensions to the process language.
- Representation of gradients of regions that are not described by a constant value but by an interval. This requires imposing a direction on the variation of values within a region and developing techniques for compositions of gradient fields.
- Automatic generation of qualitative models from sparse metric data in the form of scattered observation points. Triangulation techniques and Voronoi diagrams combined with heuristics are a possible solution. Preliminary results have been presented in (Lundell 1994).
- Extending the model with ordinal information on the sizes of spatial features. This would make it possible to model processes at different scales, and also to eliminate some of the spatial ambiguities. This technique has been used in a qualitative model of gradient flow presented in (Lundell 1995).

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