# Inference of Local Rainfall Using Qualitative Reasoning

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# Abstract

A new method, which is based on Qualitative Reasoning, for severe rainfall forecasting which occurs on a relatively small scale is presented in this paper. First, the overview of existing numerical weather forecasting methods are shown with their relating scales. Second, qualitative inference of local wind is presented. In this scheme, the partial differential equation is modeled qualitatively and diagnostically. Finally, a qualitative cumulus model is discussed. This cumulus model is designed to simulate cumulus development process with consideration of cloud microphysical processes and is expected to infer the time series variation and distribution of severe rainfall.

### Introduction

Weather forecasting is important for preventing disasters caused by extreme weather such as severe rainfall, severe drought, severe heat and severe snow. In Japan, severe rainfall is one of the most well-known causes of disaster. Here, a disaster which occurs from severe rainfall in northern region of Japan last year is shown as an example to discuss the characteristics of Japanese flood: A severe rainfall caused the flood in the northern region of Japan from July 11 to July 12, 1995. During this flood a life was lost and 68 houses were swept away (Asida & Tsujimoto 1996). Figure 1 shows the hyetograph (time series variation of rainfall intensity) at one of the observation points along the flooded river. As shown in the figure, characteristics of the rainfall are its shortduration (about two days), its amount of rainfall (almost 200mm), and its multiple peaks. These characteristics are well-known characteristics of severe rainfall in Japan, making the forecast of severe rainfall difficult and the experience of the river authority has been the most effective source of knowledge for disaster prevention. The difficulty of forecasting severe rainfall stems from locality of rainfall caused by the unique topographical characteristics of Japan. More precisely, mountains in Japan normally consist of complex topography, while also in close proximity to large water



Figure 1: Hyetograph at Akagawa in Niigata Prefecture, JAPAN

bodies.

To break the difficulty of forecasting, the mechanism of severe rainfall is investigated and observation systems for rainfall are continuously being developed. Then weather forecasting over a relatively large scale is accomplished. However, severe rainfall in Japan typically occurs suddenly over relatively small areas making forecasting of rainfall over small areas difficult. Rainfall in small areas, where many people live near easily flooded rivers, can cause severe disasters.

Qualitative Reasoning (QR), we think, is an effective way to forecast the atmospheric phenomena found typically on relatively small scales. This paper shows the forecasting method of severe rainfall over relatively small areas using Qualitative Reasoning.

# **Forecasting a Severe Rainfall**

#### Scale of Atmospheric Phenomena

The scale of weather which is related to the methods of observation and forecast is explained in Table 1. Atmospheric phenomena are classified into a number of scales and are thought to be effected by factors averaged in each scale. The largest scale is called the synoptic scale having more than a square width of 2000km. The meso scale is the middle

Scale	Phenomena	Observation	Forecasting System
Synoptic, Meso-α Scale (200 km - )	Typhoon Front	Satellite observation 1. Distribution of Cloud and Snow 2. Distribution of tem- perature of see and land surface Upper-air observation 1. Profile of tempera- ture, humidity and pressure 2. Wind speed and di- rection	Numerical Model Analysis of GPV Weather Forecasting
Meso-β Scale (20 km - 200km)	Cloud Band Cloud Cluster Rain Band	Radar 1. Rainfall area 2. Rainfall intensity	Observation Result of Rader and AMeDAS
Meso γ Scale (2 km - 20km)	Tornado Thunderstorm Cumulus, Cumulonimbus	Observation 1. at the ground level including AMeDAS 2. at the surface of the sea 3. on airplain	cal data acquisition system) Nowcast (extrapolation)

### Table 1: Classification of Atmospheric Phenomena Considering the Scale

scale and is further classified into three classes; the meso- $\alpha$  scale having a square width of 2000 - 200km, the meso- $\beta$  scale having a square width of 200 - 20km and the meso- $\gamma$  scale having less than a square width of 20 km. Typhoon and front are classified in the meso- $\alpha$  scale phenomena. Cloud band and rain band, in which a number of cumulus make a line and heavy rainfall continues, are usually classified into the meso- $\beta$  scale phenomena. The cumulus, cumulonimbus, thunderstorm and tornado are classified in the meso- $\gamma$  scale phenomena. A severe rainfall which is usually caused from cumulus, cumulonimbus or rain band is classified in the meso- $\beta$  to  $\gamma$  scale phenomena.

The upper-air observation, which observes profile of temperature, humidity and wind speed and direction at the point that is located every 250km, and meteorological satellite observation is an observation method of weather in the synoptic scale and the meso- $\alpha$  scale. The data obtained by these observation are accumulated instantaneously and is adjusted spatially and temporally uniform by using a physical diagnostic model. This adjustment is called four dimensional data assimilation and the adjusted data are used as the initial condition of the weather forecasting model.

### Numerical Weather Forecasting

Weather forecasting is usually based on the result of a numerical atmospheric model. The numerical model consists of the partial differential equations which are usually solved by the spectral method. The Japan Meteorological Agency had three types of weather forecasting model: the GSM (Global Spectral Model), which calculates world wide weather information, the ASM (Asian Spectral Model), which calculates more detailed weather phenomena than that of the GSM throughout Asia using the result of the GSM, and the JSM (Japan Spectral Model), which calculates more detailed weather phenomena than that of ASM in Japanese regions using the results of the ASM. Recently, these models have changed and the two types of weather forecasting model, GSM and RSM (Regional Spectral Model) are used.

	Scale	Grid Size	Forecasting Time
GSM	Global- Synoptic Scale	1.125° (-110km)	72hr, 192hr, 360hr
ASM	Synoptic - Meso-α Scale (350km -)	75 km	48hr
JSM	Meso Scale (150Km -)	30 km	24hr

Table 2: Outline of GPV

However, the models of three types are considered in this paper. At the same time, the result of a weather forecasting model called GPV, which means the Grid Point Value, is relayed to the river authority. Table 2 shows the data type of GPV. The GPV includes information from synoptic to the meso- $\alpha$  scale. As mentioned above, severe rainfall is often classified in the meso- $\beta$  to  $\gamma$  scale.

Weather of the meso- $\beta$  to  $\gamma$  scale is difficult to forecast numerically because of limited computational resources, stability of calculation, time and cost for computation and exactness of computation that each phenomenon of this scale, such as cloud development and local front generating, requires. The short term rainfall forecast, which aims at forecasting rainfall having a lead time of one to six or one to twelve hours in the meso- $\beta$  to  $\gamma$  scale, is usually based on the linear extrapolation of rainfall area movement and its intensity, however the linear extrapolation method is effective only when less than three hours of forecasted rainfall is required since non-linear rainfall area movement is distinctive in the frame over three hours. Forecast having a lead time of more than three hours must consider physical processes of cumulus.

According to (Sower 1966), important factors which should be considered in the physically based forecasting method of severe rainfall of the meso- $\gamma$  scale are:

- 1. initial condition of cumulus which is the main source of severe rainfall,
- 2. effect of topography on cumulus development,
- 3. cloud microphysics.

In the following section, a method to forecast the amount and location of severe rainfall with considering these factors are shown.

#### Forecasting Severe Rainfall Using QR

Inference methods, which is another forecasting method than the existing numerical methods, can be an effective forecasting method through consideration of the three important factors mentioned in the previous section and can break the computational difficulty since weather forecasters make forecast of severe rainfall by inference with taking care of these factors qualitatively. Therefore, forecasting severe rainfall by using QR is reasonable since QR is the most effective when inferring physical phenomena.

Another advantage of QR for forecasting severe rainfall is the possibility of real time explanation of inference processes to the river authority when emergency control of dams and levees are needed. Real time explanation increases the reliance of a forecasting system since the river authority decides control policy only after a forecasting system output is understood. A numerical forecasting method needs an analysis for explanation of the results; however it is difficult for the river authority to analyze these numerical results during an emergency.

Therefore, we first show a way to obtain vertical wind occurrence by solving the continuous equation qualitatively. In essence, this method creates a qualitative model from the existing numerical model. In this method, we propose a diagnostic answer to the problem of the partial differential equation (the continuous equation) by transforming the coordinate systems. Second, the concept of Qualitative Cumulus Model is shown as a method to obtain the time series variation and distribution of severe rainfall.

# **Inference of Vertical Wind Occurrence**

## **Overview of the Inference**

There are three factors which are important for initial condition of cumulus; the stability of atmosphere, the updraft caused by topography and the water vapor supply, as shown in figure 2. The stability of atmosphere is determined from profiles of temperature and humidity. Unstable atmosphere rise due to its buoyancy, becoming cumulus. Presence of updraft acts as a trigger of cumulus and also suspends cloud particles in the air. Water vapor supply, which provides the energy for cumulus, is determined by amount of water vapor and horizontal wind strength. Water vapor provides energy for cumulus, which is created by these three factors and continues to develop, ultimately causing severe rainfall.

As indicated in figure 2, inferring updraft is one of the important factors for forecasting severe rainfall in a small area. Therefore, the inference method of updraft occurrence caused by topography averaged in the meso- $\gamma$  scale is designed and implemented. The inference flow is:

1) obtain speed and direction of horizontal wind of the meso- $\alpha$  scale from GPV data as well as the topographic information of the same scale,



Figure 2: Important Factors for Forecasting Severe Rainfall in the Meso  $\beta$  to  $\gamma$ scale

- infer the horizontal movement of wind considering the effect of mountainous topography of the meso-γscale,
- 3) infer the effect of slope and the effect of convection,
- infer the vertical wind occurrence considering both effect of slope and convection.

#### **Inference Scheme**

First, the inference of horizontal movement of wind is explained. An assumption is used for representation of horizontal wind movement effected by the mountainous topography averaged in the meso- $\gamma$  scale. The assumption is that wind will blow around a mountain, which is represented by,

$$\begin{cases} [u^*] - [u] = -[K][\partial h / \partial x], [u^*] \neq -[u] \\ [v^*] - [v] = -[K][\partial h / \partial y] \end{cases}, \quad (1)$$

where, the valiables in brackets means qualitative variables, the x axis is set along the direction of wind of the meso- $\alpha$ scale, given from the numerical information, u and v represent the x and y component of horizontal wind speed of the meso- $\alpha$  scale, u\* and v\* represent the x and y component of horizontal wind speed effected by mountainous topography, h represents topography and K represents a coefficient having a positive value. Empirically, this assumption can be considered reasonable. Second, inference of vertical wind occurrence is explained. The basic equation of inferring the updraft caused by topography is based on the well-known continuous equation of wind,

$$\frac{\partial}{\partial x}(\rho_* u) + \frac{\partial}{\partial y}(\rho_* v) + \frac{\partial}{\partial z}(\rho_* w) = 0, \qquad (2)$$

where, u, v and w are wind speeds along the x-, y- and zaxis, respectively, and  $\rho_0$  is the air density. For representing the topography, a transformation of the z-axis to a saxis is used for the axis of height. The s-coordinate transformation is represented by,

$$s = \frac{z - h}{H - h},\tag{3}$$

where, h is the topographical height and H is the height of the tropopause, which is the upper boundary of the troposphere (lowest layer of the atmosphere). Equation (2) is transformed to equation (4) by introducing the *s*-coordinate,

$$\frac{\partial}{\partial s}(\rho_{s}\omega) = -\rho_{s}\left\{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right\} + \frac{\rho_{s}}{H-h}\left\{u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y}\right\}, \quad (4)$$

where,  $\omega$  is the vertical wind represented in *s*-coordinates. The first term on the right-hand side of equation (4) represents the effect of convergence, while the second term represents the direct effect of slope. To infer the occurrence of updraft, the effect of convergence and topography is inferred first, then occurrence of updraft is inferred by addition of these effects. Using equation (4), presence of updraft is qualitatively inferred since  $\omega$  at the ground level is zero. Consequently, equation (4) is transformed to qualitative equation (5),

$$[w] = -\left\{ \left[ \frac{\partial u}{\partial x} \right] + \left[ \frac{\partial v}{\partial y} \right] \right\} + \left\{ \left[ u \frac{\partial h}{\partial x} \right] + \left[ v \frac{\partial h}{\partial y} \right] \right\}, \quad (5)$$

i.e. updraft occurs when the left-hand side of equation (5) is [+] and downdraft occurs when the left-hand side of equation (5) is [-].

### **Example of Inference**

With the aid of figure 3, an example of inferring the occurrence of updraft is presented. First, the notation is explained. In figure 3, the point where updraft is inferred is marked by a triangle and the curved lines are contours of topography averaged in the meso- $\gamma$  scale. Wind averaged in the meso- $\alpha$ scale is represented by the thick arrow, U, and the wind averaged in the meso- $\gamma$  scale is represented by the thick dotted arrow,  $U^*$ . The x-axis is set in the direction of the meso- $\alpha$  scale wind, while the  $x^*$ -axis is set in the direction of the meso- $\gamma$  scale wind. The meso- $\alpha$  scale wind is given and the horizontal wind of meso- $\gamma$  scale is inferred by equation (1) in the x coordinate system. Then the occurrence of updraft, w, is inferred using equation (5) in the  $x^*$ -coordinate system.

Next, we explain the inference process in figure 3. Since in this example,  $\partial h/\partial y$  is [+] and the y-component of the meso- $\alpha$  scale wind is [0], the qualitative value of the y-component of the meso-yscale wind is inferred to be [-]. On the other hand, since  $\partial h/\partial x$  is [+] and the x-component of the meso- $\alpha$ scale wind is [+], the x-component of the meso- $\gamma$  scale wind is inferred to be smaller than that of the meso- $\alpha$  scale wind. Consequently the wind effected by the topography averaged in the meso- $\gamma$  scale is represented by the dotted thick arrow, U\*. Now, the x\*-axis is set in the same direction of the meso- $\gamma$  scale wind. The updraft is inferred by using the meso- $\gamma$  scale wind, the x\*-coordinate system and equation (5). Both  $\partial h/\partial h$  $\partial x^*$  and  $\partial h/\partial y^*$  are increasing in the x\*-coordinate system, so the topographic effect of updraft is [+]. There is no effect of convergence since the topography is strictly increasing along both the x\*-axis and the y\*-axis. Consequently, occurrence of updraft, w = [+], is inferred at the inference point.

The case for non-strictly increasing topography is explained using figure 4. This inference is the case when updraft is blowing along a valley. The meso- $\gamma$  scale wind is inferred by following the same process as was used in figure 3. To infer the effect of convergence, the meso- $\gamma$  scale wind is inferred at two points other than the inference point; one is along the  $x^*$ axis and the other is along the  $y^*$ -axis.  $\partial u^*/\partial x^*$  is inferred as [-] by comparing the wind at the inference point and the wind at the other point along the  $x^*$ -axis.  $\partial v^*/\partial y^*$  is inferred as [-]



Figure 3: An Example of Inference of Wind Effected by Topography



Figure 4: An Example of Inference of Wind Effected by Topography (a case along a valley)

by comparing the wind at the inference point and the wind at the other point along the  $y^*$ -axis. Consequently, the effect of convergence is inferred as [+]. Then the effect of topography, also inferred similarly to figure 3, is [+]. Using these two inferences, the occurrence of updraft is inferred at the inference point.

#### Scale of Topography for Inference

Obtaining  $\partial h/\partial x$  from numerical topographic data is one hurdle of this method in which the partial differential equation is solved using qualitative reasoning. Topography is obtained by grid data having mesh scales from 50m to 1km. Using 1km mesh topography data seems appropriate since the meso- $\gamma$  scale cumulus is thought to be larger than 1km. However, variation of high frequency appears in 1km mesh topography data since the height of the mesh data is not the average over the mesh scale. The low pass filter is used for removing this variation of high frequency. The equation of the low pass filter is shown from (Doswell1977) in equation (6), (7).

$$\overline{h}(x_{m}, y_{n}) = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} w(x_{m} - x_{i}, y_{n} - y_{j})h(x_{i}, y_{j})}{\sum_{i=1}^{M} \sum_{j=1}^{N} w(x_{m} - x_{i}, y_{n} - y_{j})}, (6)$$
$$w(x, y) = \exp\left(-\frac{x^{2} + y^{2}}{4\alpha}\right), (7)$$

where,  $(x_m, y_n)$  is the grit point where the filtered topography is obtained, *h* is unfiltered topography and  $\alpha$  is a parameter for the filtering scale. The 100m mesh topography is filtered and variations whose wave length is less than 1km is almost removed. Then the filtered topography is adjusted to a 1km mesh scale. Consequently, the value for  $\partial h/\partial x$  is obtained by comparing the filtered mesh data of 1km mesh scale.

The inference method of obtaining updraft caused by topography has been shown. The cumulus appearance is inferred after the other two factors in figure 2 are inferred qualitatively. Development and movement of the cumulus must be inferred to obtain the severe rainfall distribution.



Figure 5: Microphysical Prcesses of Cumulus Model

## **Qualitative Cumulus Model**

#### **General Description**

Development and movement of the cumulus have been modeled numerically. The model of cumulus development is called a microphysical model. The model of cumulus movement which is mainly caused by wind is called the dynamic model or the meteorological primitive model. The microphysical model consists of the physical processes shown in figure 5, (Oishi et al. 1994), and the meteorological primitive model consists of the equation of continuity, the momentum equations, the thermodynamic equation, the Poisson equation to solve pressure distribution.

Here, the conception of the Qualitative Cumulus Model (QCM) which simulate cumulus development processes by using Qualitative Reasoning is shown. In QCM, existence, occurrence, development and variation of qualitative variables which represent precipitation particles, wind, water vapor etc. are inferred qualitatively. This inference is based on the qualitative microphysical processes and the qualitative dynamic processes. These processes are represented by the following format; when conditions A and B are satisfied then a process C occurs and a result D is created. Inferring the processes, the QCM simulates the physical aspect of cumulus development and interaction processes of each cumulus and then the distribution of rainfall is obtained qualitatively. Consequently, QCM will provide real-time forecast of the occurrence and variation of severe rainfall from cumulus or cumuloninbus.

### Structure of the QCM

The QCM consists of the initial condition, boundary condition, the variables and microphysical processes. The initial condition includes the profile of temperature, humidity and horizontal wind speed. It is obtained from the GPV of the JSM. Using the meso- $\alpha$  scale information as an initial condition of the meso-yscale model is the most customary and practical way to start. The atmosphere, (only the troposphere is considered), is classified into four elevation types: lower layer, lower-middle layer, upper-middle layer and upper layer. This classification is based on the temperature shown in figure 6. Height of cloud base is the boundary between the lower layer and the lower-middle layer. The height of 0 °C is the boundary between the lower-middle layer and upper-middle layer. Finally, the height of -40 °C, where water does not exist as liquid under any circumstance, is the boundary between upper-middle layer and upper layer. Cloud particles exist as rain drops (relatively large water drops) in the lower layer, cloud drop (relatively small water drops) and rain drops in the lower-middle layer, ice drops (sphere shaped ice) in the upper-middle layer and ice crystals (plate shaped ice) in the upper layer.

The topographic information is used as boundary condi-



Figure 6: Classified Elevation Type in QCM

tion. The topography is obtained as the method mentioned above.

The variables consist of dynamic variables and microphysical variables. The dynamic variables consist of temperature variation, humidity variation, horizontal wind speed variation and vertical wind direction. The microphysical variables consist of the presence of precipitation particles which includes cloud drop (relatively small water drop), rain drop (relatively large water drop), ice drop and ice crystals at each elevation of atmosphere.

The microphysical processes in the QCM are nucleation, condensation, freezing, sublimation and melting. Each process is explained as follows;

- Nucleation: nucleation occurs if the stability of atmosphere is unstable, a trigger exists and enough water vapor is supplied. When nucleation occurs, cloud drops exist.
- Condensation: cloud drops develop into rain drops by condensation. Condensation occurs when enough water vapor is supplied and cloud drops exist.
- Freezing: cloud drops and rain drops freeze when they are brought into upper-middle layer.
- Sublimation: when the upper layer is saturated, sublimation occurs and ice crystals are created.
- Melting: melting occurs when ice crystals or ice drops fall into the lower-middle layer.

The remaining processes consist of dynamic and thermodynamic processes including advection, atmosphere raised by buoyancy, falling, dragging, supporting precipitation particles, divergence, convergence, suspending of ice crystals and releasing of latent heat. Each process is explained as



Figure 7: Cumulus Developing Processes Inferred by QCM

follows;

Advection: precipitation particles are blown horizontally by wind.

- Atmosphere raised by buoyancy: when temperature of an air patch is higher than temperature of the surrounding air, the air patch is raised by buoyancy and updraft occurs. This rising process continues until the level of free convection (LFC).
- Falling: precipitation particles except ice crystals fall without support from updraft.
- Dragging: if the precipitation particles fall, downdraft occurs by dragging.
- Supporting: precipitation particles are supported by updraft.
- Horizontal divergence: when updraft and downdraft occur at the same elevations, horizontal divergence occurs. Horizontal divergence also occurs when down-draft occurs at the lower layer.
- Vertical convergence: when local wind collides against each other, vertical convergence occurs updraft.
- **Suspending:** ice particles are suspending since their density is small (0.1g/cm<sup>3</sup>) and the shape of ice particles (often plate like) disturbs their fall.
- Releasing of latent heat: latent heat is released and temperature of the air patch increase when nucleation, sublimation and condensation occur.

# Forecasting Severe Rainfall Using QCM

Figure 7 illustrates the expected phenomena inferred by QCM after implementation. Expected phenomena are explained as follows: nucleation occurs and cloud drops are

created in the lower-middle layer. Temperature of the air patch increases and the air patch rises since it is buoyant. Cloud drop can be supported and then raised; the air patch rise up to the level of free convection. Cloud drops become rain drops, ice drops and ice crystals as it rises. Each precipitation particle flow horizontally by advection. As a result of advection, precipitation particles are not supported by updraft and tend to fall. The area where precipitation particles fall is where severe rainfall and downdraft occurs. Downdraft tends to diverge at the ground level. Horizontal wind that is a result of this divergence creates an updraft where it blows against the topography.

In the QCM, interaction between cumulus is represented by two phenomena; one is a trigger of cumulus and the other a combination of cumulus. Downdraft caused by dragging tends to diverge at the ground level, then horizontal wind blows. This horizontal wind might cause updraft in mountainous topography and forms another cumulus. Combination is represented by advection, which transports precipitation particles to other colliding cumulus.

Finally, the growing potential of the QCM for forecasting is discussed. The QCM must grow by obtaining qualitative and/or quantitative information of cumulus and atmospheric phenomena. To obtain such information, investigation of cumulus by using cumulus numerical models, data analysis and observations must be continued. The QCM will grow by taking the information qualitatively. Interaction processes between cumulus, except the two processes mentioned above paragraph, are most important processes for further growth of the QCM. Growth of the QCM by including the qualitative information reflects development of meteorology, (especially cumulus microphysics), for river management.

## Conclusions

Numerical methods are important to forecast and to investigate the weather for professional atmospheric scientists and forecasters. However, Qualitative Reasoning is effective to forecasting the severe phenomena which might cause disasters. In this paper, we first presented the inference method of occurrence of updraft caused by the topography averaged in the meso-y scale, which is one of the important factors needed to forecasting severe rainfall in a small area. In this method, an answer to the problem of the partial differential equations is proposed. Second, the concept of Qualitative Cumulus Model (OCM) is explained. The OCM forecasts the time series variation of distribution of severe rainfall by tracing the physical processes. Finally, we must implement the QCM must be implemented and run to discuss the possibility of forecasting rainfall in real-time while addressing the problems of the QCM.

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