An Ontological Theory of Physical Objects

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Abstract

We discuss an approach to a theory of physical objects and present a logical theory based on a fundamental distinction between objects and their *substrates*, i.e. *chunks of matter* and *regions of space*. The purpose is to establish the basis of a general ontology of space, matter and physical objects for the domain of mechanical artifacts. An extensional mereological framework is assumed for substrates, whereas physical objects are allowed to change their spatial and material substrate while keeping their identity. Besides the parthood relation, *simple self-connected region* and *congruence* (or *sphere*) are adopted as primitives for the description of space. Only three-dimensional regions are assumed in the domain. This paper is a revision and slight modification of [Borgo *et al.* 1996].

1. Introduction

Many knowledge based systems applied in the automation of engineering tasks are based on qualitative models of mechanical artifacts. Such models are often developed with a specific task in mind, and therefore only the relevant knowledge is represented. However, the high costs assŏciated their development motivate the introduction of general, task-independent ontologies, suitable to supporting very basic kinds of reasoning like those related to space, matter, and time [Neches *et al.* 1991]. In this case, it is important to consider a few simple and intuitive primitives with a good characterisation of their properties. Following this approach, we make explicit the ontological assumptions underlying the primitives and we can restrict the possible interpretations in such a way to exclude (some) non-intended models [Guarino *et al.* 1994]. The aim of this paper is to introduce and characterise, by means of logical axioms, the basic ontological distinctions needed to reason on physical objects. In our opinion, such distinctions should account for the following intuitions:

- Physical objects are located in space, can move across space; when an object moves, it occupies a different region of space. So the space occupied by an object is different from the object itself.
- Most physical objects are made of matter, but this matter is different from the object itself: when a gold ring is melted to form a kettle, a new object is created out of the same matter, and the previous object is destroyed.
- Some physical objects are immaterial (like a hole), but still they do not coincide with the space they occupy [Casati and Varzi 1994].

Our main claim regards the distinction between objects and their *substrates*, i.e. the space they occupy and the matter they are made of. The reason of this distinction lies in the different *identity criteria* of the entities involved. Consider for instance the gold ring in the example above: we recognise its concrete existence in a given situation on the basis of certain properties, like having a certain form and size, being made of a certain material, and so on. By verifying the satisfaction of these properties we are also able to recognise *the same ring* in another situation, where maybe its spatial location has changed or even a tiny piece has been lost (Fig 1). On the other hand, when focusing our attention on the matter the ring is made of, we shall use *different criteria* to recognise the existence of that particular amount of matter, which will be the same as long as no piece of it is removed, independently of any properties regarding shape, physical integrity and so on.

We are not attempting here a formal characterisation of identity criteria, which is notoriously a very difficult problem (see [Wiggins 1980, Simons 1987]) and, due to the general task we are pursuing, we don't assume any functional property. Therefore, in the following we limit ourselves to the definition of a framework in terms of admittable *spatial-material configurations*: that is, we describe the mereo-topological and morphological properties of regions, the mereological properties of matter and the relations between regions, matter and objects. This framework is conceived as a general system where peculiar identity criteria can be added for specific demands.

The above examples indicate fundamental properties of the world that our theory should be able to deal with, therefore we distinguish our entities in *objects*, *chunks of matter* and *regions of space*. These are disjointed sets of entities in our domain and their relations are characterised by a set of logical axioms. The result is a *rich* theory in terms of axioms and definitions (in the spirit of [Hayes 1985]), since the main purpose is to *convey meaning*; the theory is not therefore intended to be directly implemented in a reasoning system.

Considering the problems that a researcher has to face in finding qualitative models of mechanical artifacts and the common properties of space, matter and objects, we sustain that a general and clear theory of these properties can constitute a good basis to develop and improve knowledge representation and reasoning. At the same time, working in a task-independent theory can help in comparing and joining different approaches. This is actually an interesting aim and it is not clear in general how to face it, if this is even possible.

In the section 2 we discuss the general assumptions underlying this theory; in Section 3 we give the mereological framework adopted for substrates, while in Section 4 we present a set of axioms for the topology and morphology of regions; in section 5 we consider the domain of objects, and discuss the various relations holding between objects, matter, and space; finally, in the last section we show some (ontological) distinctions among physical objects that can be taken in this framework.

2. General Assumptions

First of all, we clearly distinguish between physical objects and their substrates.

In the case of physical objects, we limit ourselves to their properties bound to the *spatial-material configuration*, assuming that an object can be described by the set of its admittable spatial-material configurations. A particular solid cube, for instance, may be described by the class of all (roughly) cube-shaped configurations that involve the same amount of matter; it is a question of the particular identity criteria we are interested in, and so it is outside our present task to state if in this class of admitted configurations we include the case where the cube has "lost" a piece, however thin, and still it is considered to be the same cube (Fig. 1).

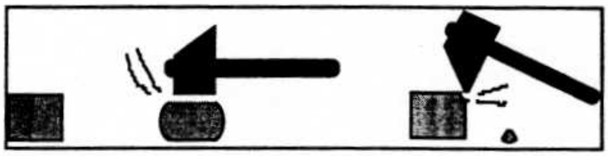

Fig 1. Still the same cube?

To concentrate on the fundamental ontological properties of substrates, we consider a simplified world where the classical properties of space hold but the space itself is not considered as a set of points. We have in mind a Newtonian notion of space where regions of space are fixed entities and they are used to recognise when a specific chunk of matter has changed its position. A reason for this distinction between space and objects comes from the desire to represent *movement*: in order to say that something has moved across space, it is natural to look for *different* regions of space occupied by *the same* object. Indeed, current AI approaches dealing with the representation of movement, like [Shanahan 1995] or [Randell and Cohn 1992, Davis 1993] postulate an ontological distinction between objects and regions for this purpose.

Our second claim regards a further distinction between objects and chunks of matter. In space, there exists a certain quantity of incompressible matter and this matter is all of one kind. Moreover, matter can assume different configurations within space. As explained above, the reason of this choice lies in the different identity criteria of the entities involved. Taking matter into account is a natural way to distinguish between a material body and a hole, or between an imaginary boundary dividing two adjacent parts of a body and a physical boundary marked by matter discontinuity. As we shall see, we use the possible spatial configurations of matter to determine the range of possible *states* of the system: the properties of objects, such as rigidity and integrity, can be expressed with reference to such a set of states.

What we propose is to carefully distinguish among four different subdomains: a set R of regions of space, a set M of chunks of matter, a set S of system states, intended as global spatial configurations of the elements of M, and a set OB of physical objects.

Regions of space can be intended as either *regular self-connected three-dimensional set of points* in a Euclidean space or mereological sums of a finite number of such regions. Following [Randell and Cohn 1992], we do not distinguish between open and closed regions. A pointless theory of space is adopted based on the primitives of parthood, strong connection and either congruence, taken from [Borgo *et al.* 1996]. We will discuss this in sections 3 and 4.

Chunks of matter are either single integral pieces of matter or finite mereological sums of such pieces. Only one kind of (incompressible) material is assumed. The set M of all chunks of matter is called *material system.*

Following [Borgo *et al.* 1996], we assume two primitive parthood relations for matter and space respectively, generating two separate mereological non-complete lattices. A *state* is seen as a parthood-preserving homomorphism, which establishes a relation between the lattices of space and matter by taking chunks of matter and returning the region they occupy. Such a homomorphism represents a possible *spatial configuration* of our material system. States are included in the domain, as current practice in situation calculus [McCarthy 1968].

Physical objects (or simply *objects*) are seen as entities related to space and matter by means of *dependence relations*, whose identity criteria are different from those of space and matter. In fact, the latter are considered *substrates*, in the sense that they have to exist (at least space, usually matter, too) in order to make possible the existence of a physical object. Moreover, the existence of substrates does not depend on the particular state we consider, while this is the case of physical objects: for example, if s is the state corresponding to a (completely) broken glass, it is plausible to assume that the glass doesn't exist in that state, while (its) matter does.

Now we will describe briefly the relations we assume in our language.

The primitive ternary predicate 'LOC' holds between spatial location, chunk of matter (or object) and state, the proposition LOCxyz is to be read "x is the location of y (which is a chunk of matter or an object) in the state z". This relation is extended to allow spatial location as the second element. Moreover, a similar relation 'MAT' is introduced, which gives the chunk of matter constituting an object in a particular state, the proposition MATxyz is to be read "x is the chunk of matter of the object y in the state z". Together, the two relations 'LOC' and 'MAT' completely specify the spatio-material behaviour of physical objects, by giving all their *admittable* spatial and material extensions. Once these two relations are given, then a physical object can be *recognised,* in the sense that a particular spatial pattern assumed by a particular amount of matter can be ascribed to a particular object. Notice that a given physical object may have no matter associated in a particular state. In particular, we define an *immaterial object* as a physical object which never has matter associated: holes and boundaries are of this kind (in qualitative models of mechanical artifacts this variety of combinations is sometimes very useful).

An important difference between substrates and physical objects regards their mereological properties. An extensional parthood relation is assumed both for regions of space and chunks of matter, such that they are always identical to the sum of their parts independently of the particular state.

For physical objects, on the other hand, the very notion of "part" becomes more problematic, since they can lose or acquire parts when the state of the system changes. We introduce therefore a notion of *contingent* part of a physical object relative to a particular state. Two distinct objects may happen to have the same contingent parts in a particular state and therefore *coincide* in that state, without being identical.

In conclusion, we model physical objects as entities depending on spatial and material substrates, these objects may or may not maintain their identity when their spatio-material properties change: our goal is to establish a logical framework able to state precisely the behaviour of such properties within a particular qualitative model.

3. Mereological Framework

We adopt in the following a standard first-order language with identity. In order to distinguish the entities of the domain we assume four unary predicates R, M, OB and S representing respectively the subdomains of *regions of space, chunks of matter, physical objects* and *states.* These are assumed as being mutually exclusive and covering the whole universe[1]:

A1. $\mathrm{R}x \veebar \mathrm{M}x \veebar \mathrm{OB}x \veebar \mathrm{S}x$

The two parthood relations for space and matter postulated in our notion of a material

[1] The symbol $\veebar$ stands for exclusive disjunction. In the whole paper, free variables are assumed to be universally quantified.

system are represented in a rather standard way, by means of a *single* binary primitive predicate 'P' restricted to hold only between *substrates of the same kind* (Pxy means "x is part of y"):

A2. $Pxy \rightarrow (Mx \wedge My) \vee (Rx \wedge Ry)$

The following axioms A3-A7, equivalent to Closed Extensional Mereology [Simons 1987, Varzi 1996], are assumed for 'P'.

A3. Pxx

A4. $Pxy \wedge Pyx \rightarrow x=y$

A5. $Pxy \wedge Pyz \rightarrow Pxz$

In the two following axioms we assume that the variables x, y and z vary on an homogeneous range:

A6. $((Mx \wedge My) \vee (Rx \wedge Ry)) \rightarrow \exists z(z=x+y)$

A7. $((Mx \vee Rx) \wedge \neg Pxy) \rightarrow \exists z(z=x-y)$
where the following definitions hold:

D1. $PPxy =_{df} Pxy \wedge \neg x=y$ *(Proper part)*

D2. $Oxy =_{df} \exists z(Pzx \wedge Pzy)$ *(Overlap)*

D3. $x+y =_{df} \iota z\forall w(Owz \leftrightarrow (Owx \vee Owy))$ *(Sum)*[2]

D4. $x-y =_{df} \iota z\forall w(Pwz \leftrightarrow (Pwx \wedge \neg Owy))$ *(Difference)*

D5. $x\times y =_{df} \iota z\forall w(Pwz \leftrightarrow (Pwx \wedge Pwy))$ *(Product)*

Notice that, due to A2, the parthood relation is only defined for substrates. In section 5 we shall see how the notion of *contingent parthood* is defined for physical objects.

It is easy to prove the followings:

T1. $(\neg Pxy \wedge (Mx \vee Rx)) \rightarrow \exists z(Pzx \wedge \neg Ozy)$ (A2; A3; A7; D4) *(strong remainder principle)*

T2. $Oxy \rightarrow \exists z(z=x\times y)$ (A2; A6; A7; D5) *(existence of product)*

[2] The 'ι' operator appearing in D3-D5 may be assumed as part of the logical vocabulary or defined contextually *à la* Russel: $\psi(\iota x\phi) =_{df} \exists y(\forall x(\phi \leftrightarrow x=y) \wedge \psi(y))$

4. Topology and Morphology of Space

4.1 Topological Level

We introduce the primitive of *simple region* (*s-region*), or region "all in one piece", defined only for regions of space and denoted by the predicate 'SR', to account for topological properties. We could introduce the common relation of (point-)connection instead of 'SR' but we follow this approach to make possible a simple and natural interpretation of our primitives, and at the same time to characterise our intended models as best as possible in order to avoid ambiguous interpretations. In particular, if we consider the case of the RCC approach [Randell and Cohn 1992], where point-connection ('C') is taken as the only primitive relation, the interpretation of the theory is not clear because the authors avoid constraining some fundamental properties of their models, like the dimension of space, and there is no unique intended interpretation for their 'C' primitive.

The intuition underlying the choice of 'SR' as primitive is bound to the notion of surface connection, and it is aimed at "explaining" connection in common-sense terms. Roughly, we want to capture the property of an object which is not made out of separated parts. We can state this considering only locations. For instance consider the (location of the) top part and the (location of the) bottom part of a piece of wood. There is a surface "in the middle" corresponding to the hypothetical cut of the wood. In this case, we say that the two parts are *surface-connected*. When this happens, for every hypothetical or real separation of the region considered, we say that that region is a simple region. Generally speaking, a simple region can be, in everyday intuition, the location of a *single* thing. This does not seem the case of line-connected or point-connected regions, since no "drop" of matter would keep the corresponding material object together. The notion of surface-connection is therefore bound to that of *physical connection*. Then, within this structure, it is easy to make a distinction between the case of two objects touching each other and the very same objects glued together.

After this informal introduction, we briefly present in the following the axioms and definitions characterising the *topological level* of our theory.

Let us first introduce some preliminary definitions:

D6. $\mathrm{PO}xy =_{\mathrm{df}} \mathrm{O}xy \wedge \neg \mathrm{P}xy \wedge \neg \mathrm{P}yx$ *(proper overlap)*

D7. $\mathrm{IP}xy =_{\mathrm{df}} \mathrm{R}x \wedge \mathrm{PP}xy \wedge \forall z\, ((\mathrm{SR}z \wedge \mathrm{PO}zx) \rightarrow \mathrm{O}z(y\text{-}x))$ *(interior part)*

D8. $\mathrm{MCP}xy =_{\mathrm{df}} \mathrm{P}xy \wedge \mathrm{SR}x \wedge \neg\exists z\, (\mathrm{SR}z \wedge \mathrm{PP}xz \wedge \mathrm{P}zy)$ *(maximally connected part)*

Notice that, according to D7, a region being l- or p-connected with a region external to x must be considered as an interior part of x. 'IP' is therefore different from the relation 'NTPP' (non-tangential proper part) defined in the RCC theory. The following axioms are assumed:

A8. $\mathrm{SR}x \rightarrow \mathrm{R}x$

A9. $(\mathrm{SR}x \wedge x=y+z) \rightarrow \exists u\, (\mathrm{SR}u \wedge \mathrm{O}uy \wedge \mathrm{O}uz \wedge \mathrm{IP}ux)$

A10. $\mathrm{R}x \rightarrow \exists y\, \mathrm{MCP}yx$

A11. $\mathrm{R}x \rightarrow \exists y\, (\mathrm{SR}y \wedge \mathrm{IP}xy)$

A8 constrains SR to regions of space. A9 captures the idea of *intimate* connection between two arbitrary halves of an s-region (Fig. 2 shows that point-connection is not enough to obtain a simple region). A10 and A11 make some minimal assumptions regarding the structure of space: every region of space has a maximally connected part and, given a region of space, there always exists a simple region of whose the first is an interior part.

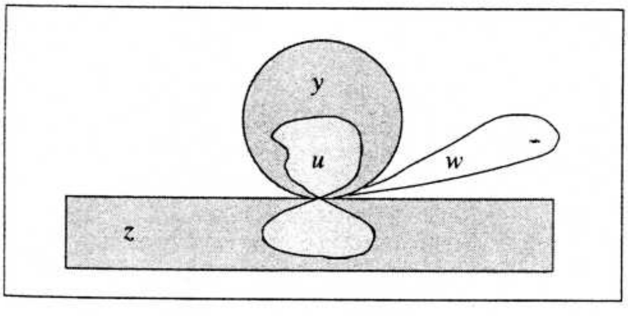

Fig 2. Why $y+z$ cannot be an s-region.

By means of SR, the relation of *strong connection* between spatial regions can be defined as follows:

D9. $\mathrm{SC}xy =_{\mathrm{df}} \exists uv\, (\mathrm{P}ux \wedge \mathrm{P}vy \wedge \mathrm{SR}(u+v))$ *(strong connection)*

Due to space limitations, we do not discuss here the consequences of the axioms presented above (for a detailed discussion of the axiomatization of space see [Borgo *et al.* 1996]); it will suffice to remark that the notion of s-connection turns out to be quite well characterised. However, we are currently not able to define l-connection and p-connection at the topological level in a satisfactory way, that is, in such a way that some unpleasant non-intended models are excluded.

Here we list some theorems which follow from the axiomatization:

T3. $\mathrm{IP}xy \wedge \mathrm{P}yz \rightarrow \mathrm{IP}xz$ (A5; D7)

T4. $\mathrm{P}xy \wedge \mathrm{IP}yz \rightarrow \mathrm{IP}xz$ (A5; D7)

T5. $\mathrm{IP}xy \wedge \mathrm{IP}xz \rightarrow \mathrm{IP}x(y\times z)$ (D5; D7)

T6. $\mathrm{IP}xz \wedge \mathrm{IP}yz \rightarrow \mathrm{IP}(x+y)z$ (D3; D7)

T7. $\mathrm{SR}x \leftrightarrow \mathrm{MCP}xx$ (D8)

T8. $(\mathrm{R}x \wedge \neg\mathrm{SR}x) \rightarrow \exists y(\mathrm{MCP}yx \wedge \mathrm{PP}yx)$ (A11; T7; D8)

T9. $\mathrm{R}x \rightarrow \exists y(\mathrm{SR}y \wedge \mathrm{IP}yx)$ (A10; T3; T8; D8)

T10. $\mathrm{IP}xy \rightarrow \exists z(\mathrm{IP}zy \wedge \mathrm{PP}xz)$ (A6; A7; T3; T6; T9)

T11. $\mathrm{SR}x \rightarrow \forall yz(y+z=x \rightarrow \mathrm{SC}yz)$ (A10; D9)

T12. $\mathrm{R}x \rightarrow (\mathrm{P}xy \leftrightarrow \forall z(\mathrm{SC}zx \rightarrow \mathrm{SC}zy))$ (A5; A7; T1; T3; T9)

T13. $\mathrm{R}x \rightarrow (x=y \leftrightarrow \forall z(\mathrm{SC}zx \leftrightarrow \mathrm{SC}zy))$ (A4; T12)

4.2 Morphological Level

The expressivity problems bound to the use of mereological and topological primitives alone is overcome by the introduction of a morphological primitive. We are forced to adopt it in any case if we want to speak of shapes, holes, edges and various morphological features. A "convex hull" primitive has been used with some success within the RCC theory, but it is probably too weak a notion for our task. A ternary alignment relation has been used in [Aurnague and Vieu 1993], but it commits to the notion of point.

Our approach makes use of simple intuition but it requires a rather new geometrical account of space. We have tried a couple of different axiomatizations: in the first case we assume a *congruence relation* between regions, denoted with 'CG', as primitive relation; in the latter we assume a primitive predicate, denoted with 'SPH', holding for spherical regions. In both cases the primitives are very intuitive and clear, but their expressivity seems different and we are presently comparing the two sets of axioms to make clear the advantages and drawbacks of the resulting systems. We show here only the general guidelines we followed to axiomatize this primitives because the full comparison of the axiomatizations would take too much space to fit in this paper.

In the case of classical geometry based on the notions of points, segments and angles, the congruent relation was first axiomatized by Hilbert [Hilbert 1902], with various simplifications thereafter. In order to take advantage from such work, we need an analogous of points in terms of regions. This analogy has been brilliantly pursued in [Tarski 1929], where a (second order) axiomatic theory taking spheres and parts as primitives was shown to be equivalent to classical geometry. Our strategy to axiomatize 'CG' is therefore the following: i) define a sphere in terms of 'P', 'SR' and 'CG'; ii) adopt Tarski's definitions related to spheres; iii) reconstruct standard axioms for congruence by exploiting the analogy between: a) points and spheres; b) segments and sums of two non-concentric spheres; c) triangles and sums of three non-concentric, non-aligned spheres. The result will be a first order theory of congruence between regions.

The crucial step here is the definition of a sphere in terms of 'P', 'SR' and 'CG', that makes it possible to link Tarski's mereo-morphological theory with our mereo-topological theory.

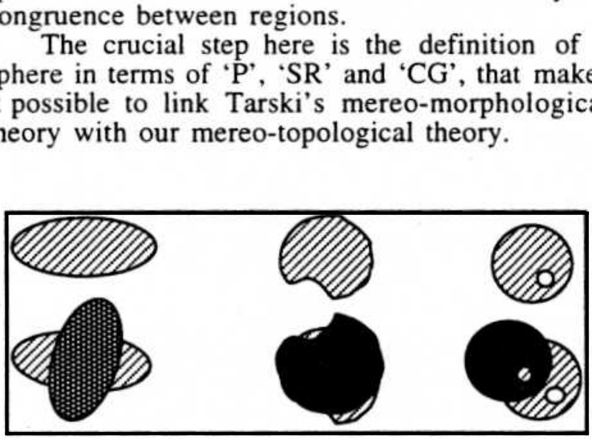

Fig 3. These regions are not spheres

D10. $\mathrm{SPH}x =_{\mathrm{df}} \mathrm{SR}x \wedge \forall y(\mathrm{CG}xy \wedge \mathrm{PO}xy \rightarrow \mathrm{SR}(x\text{-}y))$ *(sphere)*

It is easy to see that only spherical regions satisfy D10 (Fig. 3), provided that enough regions congruent to the given one exist. We force this condition through the axiomatization.

It seems to us that the notion of sphere is itself very clear and natural from an intuitive point of view. There are some advantages considering 'SPH' as a primitive predicate and it is even possible to define 'SR' using only 'P' and 'SPH'. Within this second approach our strategy to axiomatize 'SPH' is the following: i) adopt Tarski's definitions related to spheres; ii) define a notion of alignment for three spheres; iii) give a set of axioms to constrain the interpretations of all these definitions; iv) reconstruct the Euclidean axiomatization of three-dimensional space based on points using axioms based on spheres.

In both cases, the system is enriched with the predicates 'CG' and 'SPH'. We are now in the position to define l- and p-connection with the help of spheres, and then the usual notion of connection (note that SPHx implies Rx):

D11. $\mathrm{LC}xy =_{\mathrm{df}} \neg\mathrm{SC}xy \wedge \exists z(\mathrm{SPH}z \wedge \mathrm{O}zx \wedge \mathrm{O}zy \wedge \mathrm{SR}(z\text{-}x) \wedge \mathrm{SR}(z\text{-}y) \wedge \neg\mathrm{SR}(z\text{-}(x+y)))$ *(l-connection)*

In the following we use the binary relation 'CNC' which is defined and holds between concentric spheres:

D12. $\mathrm{PC}xy =_{\mathrm{df}} \neg\mathrm{SC}xy \wedge \neg\mathrm{LC}xy \wedge \exists z(\mathrm{SPH}z \wedge \forall u(\mathrm{CNC}uz \rightarrow (\mathrm{O}ux \wedge \mathrm{O}uy)))$ *(p-connection)*

D13. $\mathrm{C}xy =_{\mathrm{df}} \mathrm{SC}xy \vee \mathrm{LC}xy \vee \mathrm{PC}xy$ *(connection)*

We can easily state when a region is convex. Note that the ternary relation BTWxyz is defined and holds when x, y, z are spheres and (the center of) x is between (the centers of) y and z.

D14. $\mathrm{CONV}x =_{\mathrm{df}} (\mathrm{P}(u+v)x \wedge \mathrm{CG}uv \wedge \mathrm{CG}uw \wedge \mathrm{BTW}wuv) \rightarrow \mathrm{P}wx$ *(convex region)*

5. Matter and Physical Objects

Having stated the general characterisation of our space, we go on to the axioms for chunks of matter and physical objects. Two primitives are introduced: 'LOC' gives the spatial extension (exact location) of an individual at a particular state, 'MAT' gives its material extension at the state specified. The domain of 'LOC' is extended to include also regions, assuming that, for any state, the location of a region coincides with the region itself [Casati and Varzi 1996]. The following axioms clarify the domain of these relations:

A12. $\mathrm{LOC}rxs \rightarrow \mathrm{R}r \wedge (\mathrm{OB}x \vee \mathrm{M}x \vee \mathrm{R}x) \wedge \mathrm{S}s$

A13. $\mathrm{MAT}mxs \rightarrow \mathrm{M}m \wedge (\mathrm{OB}x \vee \mathrm{M}x) \wedge \mathrm{S}s$

A14. $\mathrm{R}x \leftrightarrow \mathrm{LOC}xxs$

A15. $\mathrm{M}x \leftrightarrow \mathrm{MAT}xxs$

As discussed in Section 2, we assume that an individual *exists* (in the ontological sense) in a state if it has a location in that state (D15). A14 and A16 make sure that regions and pieces of matter always exist; A17 states that given an object there is at least a state in which that object exists. Moreover, A16 and A17 express, respectively, the ontological dependence between matter and space and between physical objects and space.

D15. $\mathrm{EX}xs =_{\mathrm{df}} \exists r\, \mathrm{LOC}rxs$

A16. $(\mathrm{M}x \wedge \mathrm{S}s) \rightarrow \mathrm{EX}xs$

A17. $\mathrm{OB}x \rightarrow \exists s\, \mathrm{EX}xs$

The following axioms guarantee that: i) 'LOC' denotes a function from M to R with respect to the parameter s (A18); ii) such a function is injective (A19); and iii) it is an homomorphism between M and R preserving the parthood relation 'P' (A20). A21 shows how the notion of state is bound to such homomorphism.

A18. $\mathrm{LOC}r_1xs \wedge \mathrm{LOC}r_2xs \rightarrow r_1=r_2$

A19. $\mathrm{M}x_1 \wedge \mathrm{M}x_2 \wedge \mathrm{LOC}rx_1s \wedge \mathrm{LOC}rx_2s \rightarrow x_1=x_2$

A20. $\mathrm{M}x_1 \wedge \mathrm{M}x_2 \wedge \mathrm{LOC}r_1x_1s \wedge \mathrm{LOC}r_2x_2s \rightarrow$
$(\mathrm{P}x_1x_2 \leftrightarrow \mathrm{P}r_1r_2)$

A21. $(\mathrm{LOC}rms_1 \wedge \mathrm{LOC}rms_2) \rightarrow s_1=s_2$

Note that for regions of space it is possible to infer a proposition corresponding to A19, but this is not possible for objects since two different objects can be colocalized (for instance a nail and the hole where it is driven).

A22 states that, analogously to 'LOC', 'MAT' denotes a function from OB to R with respect to the parameter s. We also assume that, in the case of physical objects, their spatial location coincides with the location of the matter they are made of (A23). This last assumption may be removed if we allow for "mixed" objects (i.e., both material and immaterial), such that the region they occupy is larger than the region occupied by their matter; we shall not consider such cases here, however they could be interesting in a qualitative model of mechanical artifacts.

A22. $\mathrm{MAT}m_1xs \wedge \mathrm{MAT}m_2xs \rightarrow m_1=m_2$

A23. $\mathrm{LOC}rxs \wedge \mathrm{MAT}mxs \rightarrow \mathrm{LOC}rms$

6. Ontological Distinctions Among Physical Objects

The ontological theory developed so far turns out to be quite powerful, allowing us to establish – in a rigorous way – useful distinctions within our domain. We give here a preliminary account of some of these distinctions.

6.1 General Properties

First, it is useful to distinguish between *material* and *immaterial* objects (denoted respectively with 'MO' and 'IO') on the basis of the presence or absence of a material substrate in any state where the object exists:

D16. $\mathrm{MO}x =_{\mathrm{df}} \mathrm{OB}x \wedge (\mathrm{EX}xs \rightarrow \exists m\, \mathrm{MAT}mxs)$
(*material object*)

D17. $\mathrm{IO}x =_{\mathrm{df}} \mathrm{OB}x \wedge (\mathrm{EX}xs \rightarrow \neg\exists m\, \mathrm{MAT}mxs)$
(*immaterial object*)

We define then the notion of *contingent part* for an object in a particular state as follows. Notice that, due to A23, we exclude the case of an immaterial object being part of a material object, and vice-versa. Objects being contingent parts of another object in any state are called *essential parts* of that object.

D18. $\mathrm{CP}xys =_{\mathrm{df}} ((\mathrm{IO}x \wedge \mathrm{IO}y) \vee (\mathrm{MO}x \wedge \mathrm{MO}y)) \wedge$
$\mathrm{LOC}uxs \wedge \mathrm{LOC}vys \wedge \mathrm{P}uv$
(*contingent part*)

D19. $\mathrm{ESP}xy =_{\mathrm{df}} (\mathrm{IO}y \vee \mathrm{MO}y) \wedge \mathrm{EX}ys \rightarrow \mathrm{CP}xys$
(*essential part*)

We say that two objects *coincide* in a state if they have the same contingent parts in that state. Notice that two objects can be *constantly* *coincident* in all states without being identical:

D20. CCDxys =df $\forall z$ (CP$zxs \leftrightarrow$ CPzys) (*coincidence*)

Finally, the notion of *rigidity* for physical objects can be easily defined as follows:

D21. RIGx =df (IO$x \vee$ MOx) $\wedge$ (LOC$uxs_1 \wedge$ LOC$vxs_2 \rightarrow$ CGuv) (*x is a rigid object*)

6.2 Boundaries and Granularity

Boundaries are introduced in our framework in such a way to avoid relying on their classical mathematical definition. We adopt a definition more akin to common-sense intuition, where surfaces and edges are thought of as concrete entities, and granularity considerations are invoked. We can easily introduce a notion of *granularity* within our system by fixing a particular sphere g, and defining a *granule* of granularity g as follows:

D22. Gxg =df SPH$g \wedge$ CGxg (*x is a granule*)

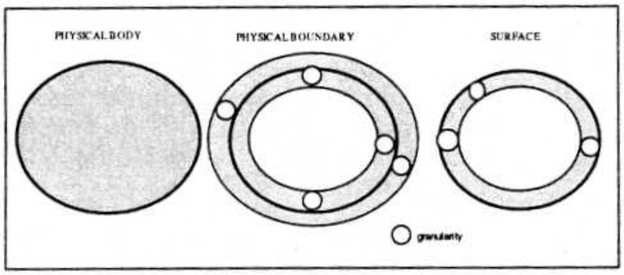

Fig 4. Boundary and surface of a physical body.

Now we can "approximate" the mathematical notion of the boundary of a region by means of a suitably thin region overlapping the "real" boundary (Fig. 4)

D23. SBxyg =df R$x \wedge$ R$y \wedge \forall z$ (P$zx \leftrightarrow \forall w$ (P$wz \rightarrow$ $\exists u$ (Gug $\wedge$ O$uw \wedge$ POuy)))

(*x is the spatial boundary of y wrt g*)

In the case of physical objects, boundaries are not intended as regions, but as *immaterial objects* always overlapping the "real boundary" as the state changes. They are called in this case *physical boundaries*.

D24. PBxyg =df $\forall s$(EX$ys \leftrightarrow$ EXxs) $\wedge$ IO$x \wedge$ OBy $\wedge$((LOC$uxs \wedge$ LOCvys) $\rightarrow$ SBuvg)

(*x is the physical boundary of y wrt g*)

Now the notion of the *surface* (or "skin") of a physical object can be defined as follows:

D25. SURFxyg =df $\forall s$(EX$ys \leftrightarrow$ EXxs) $\wedge$ ESPxy $\wedge$ ((LOC$uxs \wedge$ LOC$vys \wedge$ SBwvg) $\rightarrow u=v \times w$)

(*x is the surface of y wrt g*)

Many other useful distinctions can be made, in particular we are able to distinguish between so-called "fiat" and "bona-fide" boundaries depending on the presence of matter discontinuity [Smith 1994], and between contact and material connection among physical bodies on the basis of the morphological properties of the boundary between them, at a given granularity.

7. Conclusion

We have presented a general axiomatic system in first order logic aimed to give a framework to develop qualitative models of mechanical artifacts. The system is adapted to describe a good variety of objects and at the same time it constrains the properties of the substrate, namely space and matter.

Further developments can be made to enrich the theory so that it can deal with compressible matter and with matter of different kinds. We would like to improve the temporal aspects of this approach and generalise the notion of object to include mixed objects, i.e. objects which have both material and immaterial parts such as the sum of a glass and its hole (the part which can be filled up).

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