Limitations Imposed by the Sign-Equality Assumption in Qualitative Simulation

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Abstract

The second derivative *sign-equality assumption* has to be made for the derivation of expressions for higher-order derivatives of chattering system variables in the qualitative simulation of models containing monotonic function constraints. Kuipers *et al.* [1] established this method and showed that prediction failure is possible when the sign-equality assumption is utilized. We present a closer examination of the unwarranted modeling restrictions entailed by this assumption in connected tank systems. We show that all the systems which are validly considered in simulations with HOD constraints necessarily contain tanks with quite atypical shapes. We conclude that the chattering and spurious behavior elimination problem for systems of connected tanks with reasonable shapes is still open.

Introduction

Kuipers *et al.* [1] presented a method of utilizing higher-order derivative information to eliminate chattering behavior in qualitative simulation. As explained clearly in [1], this method sacrifices soundness (i.e. the ability to predict all qualitatively distinct behaviors that might be exhibited by all real systems corresponding to the input model) if monotonic function constraints are included in the system model. The reason for this is the sign-equality assumption that has to be made at certain points during the simulation for obtaining expressions for the second derivatives of the chattering variables. Each instance of the sign-equality assumption imposes a constraint which is not present in the input on one of the functional relationships in the system model. Behaviors predicted by the simulator have to satisfy these additional constraints as well as the original model-imposed ones, and so some genuine system behaviors that do not satisfy these unwarranted constraints are missed.

In this paper, we present a closer examination of the subsets of systems that are represented by the additionally constrained models created by the application of the sign-equality assumption. After obtaining descriptions of these soundly simulated systems, we conclude that the sign-equality assumption restricts the scope of validly considered systems too severely, at least in the particular domain of liquid flow through connected tanks.

QSIM with HOD Constraints: An Overview

The definitive reference on the QSIM representation and algorithm is [2]. In the following, we use the notation of Chapter 10 of [2], which supersedes [1]. Some key notational items are as follows:

For any real-valued expression E, [E] denotes the sign (–, 0, or +) of E.

For a system variable $x(t)$,

qdir(x) and [x'] both denote the sign of $\frac{d}{dt}x$,

sd2(x) and [x''] both denote the sign of $\frac{d^2}{dt^2}x$, and,

sd3(x) and [x'''] both denote the sign of $\frac{d^3}{dt^3}x$.

The symbols *inc*, *std*, and *dec* mean –, 0, and +, respectively, when they are used as the value of a *qdir* term.

The constraint $M^+(x,y)$ means that there is a reasonable function f mapping values of x to values of y such that f' is greater than zero throughout its domain.

Qualitative simulation with HOD (higher-order derivative) constraints starts with a stage in which the variables likely to chatter (i.e. cause an intractable simulation that branches on each of their possible "twists and turns") are identified by an automatic examination of the input model. A variable may chatter if its derivative is not constrained in the model.

The second stage involves the derivation of algebraic expressions for the second- and third-order derivatives of the variables identified in the first stage. The aim is to obtain expressions which consist solely of terms that are contained within the vocabulary provided by the input model, so that their signs can be evaluated using the qualitative information available during the simulation. A set of transformation rules is used in a search process to obtain these expressions. Here are a few example transformation rules:

$$sd2(x) \text{ where } constant(x) \quad \rightarrow \quad 0$$

$$sd2(x) \text{ where } y = \frac{d}{dt}x \quad \rightarrow \quad qdir(y)$$

$$sd2(x) \text{ where } M^{+}(x,y) \quad \rightarrow \quad sd2(y)$$

All the transformation rules are validity-preserving at critical points of chattering variables, with the exception of the last one shown above, which embodies an assumption about the monotonic relation between variables x and y. Only if this *sign-equality assumption* is correct, that is, if

$$M^{+}(x,y) \quad \Rightarrow \quad sd2(x,t_i) = sd2(y,t_i)$$

where t_i is such a critical point, will an expression obtained by using this transformation be valid.

After the derivation of the expressions for the higher-order derivatives of the chattering variables, qualitative simulation of the system commences. During the simulation, whenever a newly generated time-point state contains a value $qdir(x) = 0$ for a chattering variable x, the signs of the related higher-order derivative expressions are evaluated and the appropriate HOD constraints are applied to provide an additional filter that may eliminate that state from the tree. (The HOD constraint actually operates twice for each critical point, see [2] for the details.)

Cascaded Tanks of Restricted Shapes

Clearly, each time an expression whose derivation included the sign-equality assumption is used for a HOD filter during simulation, the reasoner is making an unwarranted assumption about the input model, eliminating a subset of the systems that are representable by it from consideration. In this section, we will examine the restrictions of this kind imposed on the shapes of the tanks in the two- and three-tank cascade systems presented in [1] by tracing the simulations.

The Two-Tank Cascade

This system (Fig. 1) has a constant inflow of liquid into the tank at the top. The explanations of the variables are given in Table 1. The model is

$$netflowA = inflow - outflowA$$

$$netflowB = outflowA - outflowB.$$

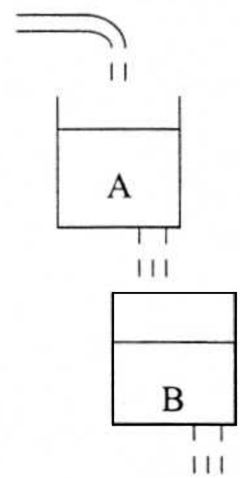

Figure 1: The two-tank cascade

Table 1. Variables in the two-tank cascade

Name	Explanation
inflow	constant inflow to tank A
amtA	mass of liquid in tank A
outflowA	outflow from tank A; this is an M^{+} function of *amtA*: *outflowA* = *f*(*amtA*)
netflowA	"net" flow into tank A; *amtA*'s time derivative
amtB	mass of liquid in tank B
outflowB	outflow from tank B; this is an M^{+} function of *amtB*: *outflowB* = *g*(*amtB*)
netflowB	"net" flow into tank B; *amtB*'s time derivative

Variable *netflowB* is identified as a candidate for chattering at the first stage, and so an expression for *sd2(netflowB)*, which will be evaluated whenever *qdir(netflowB)* equals zero in a newly generated time-point state during the simulation, is derived as follows:

$$\begin{aligned} sd2(netflowB) &= sd2(outflowA) - sd2(outflowB) \\ &= sd2(amtA) - sd2(amtB) \\ &= qdir(netflowA) - qdir(netflowB) \\ &= qdir(netflowA) \end{aligned}$$

Note that the sign-equality assumption is made twice in this derivation, in order to replace *sd2(outflowA)* by *sd2(amtA)*, and *sd2(outflowB)* by *sd2(amtB)*. Exactly what restriction this will impose on the functions *f* and *g* will be understood when the obtained expression is utilized during the simulation.

Simulation commences (Table 2,) and arrives at time point t_1, where a candidate state has *qdir(netflowB)* = 0, (Table 3) and so the HOD constraint has to be put to use. At this moment, the aforementioned assumptions

$$\begin{aligned} sd2(outflowA, t_1) &= sd2(amtA, t_1), \text{ and} \\ sd2(outflowB, t_1) &= sd2(amtB, t_1) \end{aligned}$$

have to hold.

Table 2. First two states of two-tank cascade behavior

time	t_0	(t_0, t_1)
inflow	in*, std	in*, std
amtA	0, inc	(0,∞), inc
outflowA	0, inc	(0,∞), inc
netflowA	(0,∞), dec	(0,∞), dec
amtB	0, std	(0,∞), inc
outflowB	0, std	(0,∞), inc
netflowB	0, inc	(0,∞), inc

Table 3. A candidate state considered for t_1 in the two-tank cascade simulation

inflow	in*, std
amtA	(0,∞), inc
outflowA	(0,∞), inc
netflowA	(0,∞), dec
amtB	(0,∞), inc
outflowB	(0,∞), inc
netflowB	nB*, std

Let us examine the effects of these on the functions *f* and *g*.

As explained in [2], the definition of the monotonic function *f* can be differentiated twice to get

$$outflowA''(t) =$$

$$f'(amtA(t)).amtA''(t) + f''(amtA(t)).(amtA'(t))^2.$$

Now, we know that $[amtA''(t_1)] = [-]$ (it is simply the sign of the derivative of $amtA'$, i.e. *qdir(netflowA)* in Table 3,) and $[amtA'(t_1)] = [+]$, and we are assuming that $[amtA''(t_1)]$ is equal to $[outflowA''(t_1)]$. Using the fact that $f' > 0$, we can manipulate the above equation for $outflowA''(t)$ to obtain information on the requirements about *f* at the point $amtA(t_1)$:

$$f''(amtA(t_1)) < -\left(\frac{f'(amtA(t_1)).amtA''(t_1)}{(amtA'(t_1))^2}\right).$$

All nonpositive values of f'', and the sufficiently small positive ones, satisfy this condition. See [2, p. 256] for a system which violates this restriction.

The restriction imposed on *g* is more serious. Consider the equation

$$outflowB''(t) =$$

$$g'(amtB(t)).amtB''(t) + g''(amtB(t)).(amtB'(t))^2.$$

At t_1, it is known that $amtB'' = 0$, $amtB' > 0$, and we want $outflowB''$ to be zero too, because of the sign-equality assumption. Inserting those values into the above equation for $outflowB''(t)$, we obtain

$$g''(amtB(t_1)) = 0.$$

This is a serious restriction, as will be explained later.

At time point t_2, another state (Table 4) with a critical point for *netflowB* is considered by the algorithm. The HOD constraint is used to eliminate this state, thus adding

$$f''(amtA(t_x)) < -\left(\frac{f'(amtA(t_x)).amtA''(t_x)}{(amtA'(t_x))^2}\right)$$

and

$$g''(amtB(t_x)) = 0$$

to the list of unwarranted restrictions on the functions *f* and *g*.

Table 4. A candidate state considered for t_2 in the two-tank cascade simulation

inflow	in*, std
amtA	(0,∞), inc
outflowA	(0,∞), inc
netflowA	(0,∞), dec
amtB	(0,∞), inc
outflowB	(0,∞), inc
netflowB	nB2*, std

(Note that we do not use the time index t_2, but rather another symbol t_x in the formulae above. Since the proposed state at t_2 was eventually eliminated, we are not entitled to call that time point "t_2". On the other hand, the new restriction formulae certainly hold at a time point $t_x > t_1$, otherwise the proposed state would not have been eliminated. We are justified to add any formulae stemming from HOD constraints "activated" during simulation to our list.)

Let us now examine what these conditions mean for the shape of tank B. In addition to the usual information about M^+-type functions, we know that g's second derivative has the value zero for at least two distinct values of *amtB*. (This is obvious, since *amtB* is increasing all through the simulation.) It is easy to show that g'' is always negative for a tank whose cross-sectional area does not change as a function of vertical position. So tank B in our example cannot be, say, an upright cylinder.

g'' has the value zero for tanks with "tall thin stacks" on top (like the one in the prediction failure example in [2]) only when the level of liquid reaches the base of the stack, and then only if the "curving" of the wall of the tank at that level satisfies certain conditions. So our tank B can have a structure as in Fig. 2. The simulation results for the two-tank cascade (Table 5) would then reflect the case where *netflowB* reaches its critical point exactly when the liquid level in tank B reaches L1. For all cases where the level is not at such a bend at t_1, Table 5 is not a mathematically justified representation of the behavior of this tank.

If we wish tank B's shape to be as "regular" as possible given these restrictions, we can assume that $g'' = 0$ all through a finite interval containing the points $amtB(t_1)$ and $amtB(t_x)$. Let us examine what such an assumption would mean for a monotonic function $l(amt)$ which relates the amount of liquid in the tank to the liquid level.

Under the usual simplifying assumptions applicable to such systems,

$$g(amt) = C.(l(amt))^{\frac{1}{2}},$$

$$g'(amt) = \frac{C}{2}.(l(amt))^{\frac{-1}{2}}.l'(amt), \text{ and,}$$

$$g''(amt) =$$

$$\frac{C}{2}.\left(\frac{-1}{2}(l(amt))^{\frac{-3}{2}}.(l'(amt))^2 + (l(amt))^{\frac{-1}{2}}.l''(amt)\right),$$

where C is a constant.

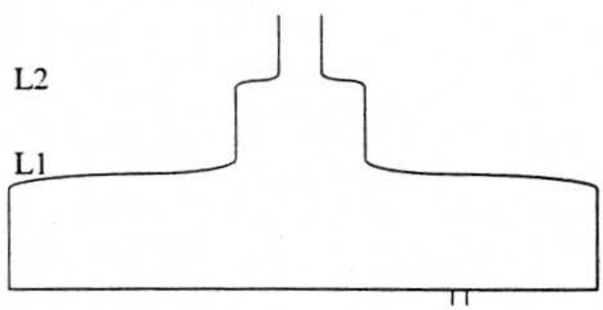

Figure 2: A possible shape for tank B

Table 5. The output of QSIM (using HOD constraints) for the two-tank cascade

time	t_0	(t_0, t_1)	t_1	(t_1, t_2)	t_2
inflow	in*, std	in*, std	in*, std	in*, std	in*, std
amtA	0, inc	(0,∞), inc	(0,∞), inc	(0,∞), inc	aA*, std
outflowA	0, inc	(0,∞), inc	(0,∞), inc	(0,∞), inc	oA*, std
netflowA	(0,∞), dec	(0,∞), dec	(0,∞), dec	(0,∞), dec	0, std
amtB	0, std	(0,∞), inc	(0,∞), inc	(0,∞), inc	aB*, std
outflowB	0, std	(0,∞), inc	(0,∞), inc	(0,∞), inc	oB*, std
netflowB	0, inc	(0,∞), inc	nB*, std	(0,nB*), dec	0, std

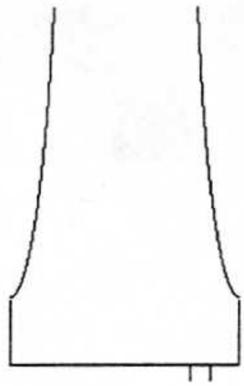

Figure 3: Another possible shape for tank B

For g'' to be zero, the sum in the parentheses has to evaluate to zero, which gives us the differential equation

$$l''(amt) = \frac{1}{2}.\frac{(l'(amt))^2}{l(amt)}$$

whose solution is

$$l(amt) = (C_1.amt + C_2)^2$$

Since $l(0) = 0$, $C_2 = 0$, and

$$l(amt) = C_3.amt^2,$$

where C_3 is a positive constant.

Tank "segments" with this property have $g'' = 0$. It is easy to see that the cross-sectional area of such a tank segment at vertical position L is inversely proportional to the square root of L. Fig. 3 is thus a representative of the set of most likely candidates for the side view of tank B, with the proviso that $amtB(t_1)$ corresponds to a level in the "thinning" segment. Fig. 4 depicts some families of shapes overruled by the sign-equality assumption for this tank.

The Three-Tank Cascade

This system (Table 6) is obtained by placing another tank named C under the two-tank cascade. The model is:

netflowA = *inflow* - *outflowA*

netflowB = *outflowA* - *outflowB*

netflowC = *outflowB* - *outflowC*

There are two variables (*netflowB* and *netflowC*) which chatter "independently" from each other, so HOD expressions are derived for both of these. The sign-equality assumptions embedded in the derivations are

sd2(*outflowA*) = *sd2*(*amtA*),

sd2(*outflowB*) = *sd2*(*amtB*), and,

sd2(*outflowC*) = *sd2*(*amtC*).

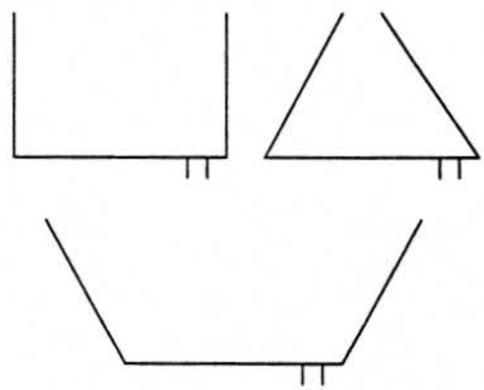

Figure 4: Forbidden shapes for tank B

Table 6. Variables in the three-tank cascade

Name	Explanation
inflow	constant inflow to tank A
amtA	mass of liquid in tank A
outflowA	outflow from A; this is an M^+ function of *amtA*: *outflowA* = *f*(*amtA*)
netflowA	"net" flow into A; *amtA*'s time derivative
amtB	mass of liquid in tank B
outflowB	outflow from B; this is an M^+ function of *amtB*: *outflowB* = *g*(*amtB*)
netflowB	"net" flow into B; *amtB*'s time derivative
amtC	mass of liquid in tank C
outflowC	outflow from C this is an M^+ function of *amtC*: *outflowC* = *h*(*amtB*)
netflowC	"net" flow into C; *amtC*'s time derivative

Table 7. The output of QSIM (using HOD constraints) for the three-tank cascade

time	t_0	(t_0, t_1)	t_1	(t_1, t_2)	t_2	(t_2, t_3)	t_3
inflow	in*, std	in*, std	in*,std	in*, std	in*, std	in*, std	in*, std
amtA	0, inc	(0,∞),inc	(0,∞),inc	(0,∞), inc	(0,∞), inc	(0,∞), inc	aA*,std
outflowA	0, inc	(0,∞),inc	(0,∞),inc	(0,∞), inc	(0,∞), inc	(0,∞), inc	oA*,std
netflowA	(0,∞),dec	(0,∞),dec	(0,∞),dec	(0,∞), dec	(0,∞), dec	(0,∞), dec	0,std
amtB	0, std	(0,∞),inc	(0,∞),inc	(0,∞), inc	(0,∞), inc	(0,∞), inc	aB*,std
outflowB	0, std	(0,∞),inc	(0,∞),inc	(0,∞), inc	(0,∞), inc	(0,∞), inc	oB*,std
netflowB	0, inc	(0,∞),inc	nB*,std	(0,nB*),dec	(0,nB*),dec	(0,nB*),dec	0,std
amtC	0, std	(0,∞),inc	(0,∞),inc	(0,∞), inc	(0,∞), inc	(0,∞), inc	aC*,std
outflowC	0, std	(0,∞),inc	(0,∞),inc	(0,∞), inc	(0,∞), inc	(0,∞), inc	oC*,std
netflowC	0, std	(0,∞),inc	(0,∞),inc	(0,∞), inc	nC*, std	(0,nC*),dec	0, std

During the simulation, the HOD constraint is employed several times. Just like in the two-tank cascade, tank A's shape is the one which is the least severely affected by the assumptions. The conditions imposed on f are the same as described in the previous section, and are satisfied by a wide variety of tank shapes, including cylindrical ones.

The restrictions imposed on h'', and hence on the shape of tank C are similar to what happened to tank B in the previous subsection. The applied HOD constraints specify

$$h''(amtC(t_{\text{rise}})) = 0,$$

$$h''(amtC(t_2)) = 0, \text{ and}$$

$$h''(amtC(t_{\text{fall}})) = 0,$$

where $t_{\text{rise}} < t_2 < t_{\text{fall}}$ (t_2 is as defined by the unique simulation result in Table 7,) and, clearly, $amtC(t_{\text{rise}}) < amtC(t_2) < amtC(t_{\text{fall}})$.

The middle tank (B) in the cascade undergoes the greatest number of restrictions. At a candidate state for t_1 (Table 8,) the general equation for $outflowB''(t)$ and the facts that $[amtB''] = [+]$, $[amtB'] = [+]$, and $[outflowB''] = [amtB'']$ is assumed yield the condition

$$g''(amtB(t_x)) > -\left(\frac{g'(amtB(t_x)).amtB''(t_x)}{(amtB'(t_x))^2}\right)$$

which is satisfied for all nonnegative and only some negative values of g''.

Another candidate state for t_1 (Table 9) necessitates *sd3(netflowC)* to be evaluated. During this expression's derivation, the expressions for both *sd2(netflowB)* and *sd2(netflowC)* are used. These imply both

$$g''(amtB(t_y)) > -\left(\frac{g'(amtB(t_y)).amtB''(t_y)}{(amtB'(t_y))^2}\right)$$

and

$$g''(amtB(t_y)) = 0,$$

so we add the latter, more restrictive equation to our list.

Table 8. A candidate state for t_1 in the three-tank cascade simulation

inflow	in*, std
amtA	(0,∞), inc
outflowA	(0,∞), inc
netflowA	(0,∞), dec
amtB	(0,∞), inc
outflowB	(0,∞), inc
netflowB	(0,∞), inc
amtC	(0,∞), inc
outflowC	(0,∞), inc
netflowC	nC*, std

Table 9. Another candidate state for t_1 in the three-tank cascade simulation

inflow	in*, std
amtA	(0,∞), inc
outflowA	(0,∞), inc
netflowA	(0,∞), dec
amtB	(0,∞), inc
outflowB	(0,∞), inc
netflowB	nB*, std
amtC	(0,∞), inc
outflowC	(0,∞), inc
netflowC	nC*, std

Table 10. A candidate state for t_3 in the three-tank cascade simulation

inflow	in*, std
amtA	(0,∞), inc
outflowA	(0,∞), inc
netflowA	(0,∞), dec
amtB	(0,∞), inc
outflowB	(0,∞), inc
netflowB	(0, nB*), dec
amtC	(0,∞), inc
outflowC	(0,∞), inc
netflowC	nC2*, std

Finally, a candidate state for t_3 (Table 10) also gets eliminated by the HOD constraint, adding

$$g''(amtB(t_z)) < -\left(\frac{g'(amtB(t_z)).amtB''(t_z)}{(amtB'(t_z))^2}\right)$$

to what we know about g''.

The "simplest" shape for tank B that would be compatible with these restrictions would again be a "linear" tank (thus called because g is a linear function when the liquid level is in the "thinning" region) such as that of Figure 3. However, the possibility that *netflowB* reaches its maximum while the liquid level has not reached the thinning region would again be overlooked by the simulator.

Ruling out a "Smooth" Tank

For another example of the theoretical problems caused by the sign-equality assumption, consider the system of Table 11, which is presented in p. 266 of [2] as a specimen to be simulated using HOD constraints. This is a U-tube with constant external inflow to the first tank and a hole at the bottom of the second tank.

The model is

$$prdiffAB = pressureA - pressureB$$
$$netflowA = inflow - flowAB$$
$$netflowB = flowAB - outflowB.$$

Table 12 shows the prediction of QSIM with HOD constraints for this system where *inflow* is positive.

Table 11. Variables in the open-ended U-tube

Name	**Explanation**
inflow	constant inflow to tank A
amtA	mass of liquid in tank A
pressureA	pressure at the bottom of A; this is an M^+ function of *amtA*: $pressureA = p_a(amtA)$
netflowA	"net" flow into A; *amtA*'s time derivative
amtB	mass of liquid in tank B
pressureB	pressure at the bottom of B; this is an M^+ function of *amtB*: $pressureB = p_b(amtB)$
outflowB	outflow from B; this is an M^+ function of *amtB*: $outflowB = g(amtB)$
netflowB	"net" flow into B; *amtB*'s time derivative
prdiffAB	pressure difference between the tanks
flowAB	flow through pipe from A to B; this is an M^+ function of *prdiffAB*: $flowAB = f(prdiffAB)$

Table 12. The behavior of the open-ended U-tube

time	t_0	(t_0, t_1)	t_1	(t_1, t_2)	t_2
inflow	in*, std	in*, std	in*,std	in*, std	in*, std
amtA	0, inc	(0,∞),inc	(0,∞),inc	(0,∞), inc	aA*, std
pressureA	0, inc	(0,∞),inc	(0,∞),inc	(0,∞),inc	pA*, std
netflowA	(0,∞), dec	(0,∞), dec	(0,∞), dec	(0,∞), dec	0, std
amtB	0, std	(0,∞),inc	(0,∞),inc	(0,∞),inc	aB*, std
pressureB	0, std	(0,∞),inc	(0,∞),inc	(0,∞),inc	pB*, std
outflowB	0, std	(0,∞),inc	(0,∞),inc	(0,∞),inc	oB*, std
netflowB	0, inc	(0,∞),inc	nB*, std	(0,nB*), dec	0, std
prdiffAB	0, inc	(0,∞),inc	(0,∞),inc	(0,∞),inc	pdAB*, std
flowAB	0, inc	(0,∞),inc	(0,∞),inc	(0,∞),inc	fAB*, std

netflowB is one of the potentially chattering variables of this system. Let us consider the derivation of the *sd2* term for it:

$$\begin{aligned} sd2(netflowB) &= sd2(flowAB) - sd2(outflowB) \\ &= sd2(prdiffAB) - sd2(amtB) \\ &= sd2(prdiffAB) - qdir(netflowB) \\ &= sd2(prdiffAB) \\ &= sd2(pressureA) - sd2(pressureB) \\ &= sd2(amtA) - sd2(amtB) \\ &= sd2(amtA) - qdir(netflowB) \\ &= sd2(amtA) \\ &= qdir(netflowA) \end{aligned}$$

This derivation involves the sign-equality assumption for two *different* functions corresponding to tank B. Take $Qstate(t_1)$ in Table 12. Since $qdir(netflowB) = 0$, the HOD constraint is utilized. The restriction imposed on g is

$$g''(amtB(t_1)) = 0 .$$

The restriction imposed on p_b at the same value $amtB(t_1)$ is

$$p_b''(amtB(t_1)) = 0 .$$

Let us see what this means. We will again use the function $l(amt)$ mapping amounts to liquid levels. In the previous section, we had established that

$$g''(amt) =$$

$$\frac{C}{2}.\left(\frac{-1}{2}\big(l(amt)\big)^{\frac{-3}{2}}.\big(l'(amt)\big)^2 + \big(l(amt)\big)^{\frac{-1}{2}}.l''(amt)\right).$$

The pressure is a linear function of the liquid level, hence

$$p_b(amt) = \mathrm{K}.l(amt),$$

$$p_b'(amt) = \mathrm{K}.l'(amt), \text{ and,}$$

$$p_b''(amt) = \mathrm{K}.l''(amt),$$

where K is a positive constant.

Knowing $p_b''(amtB(t_1)) = 0$, we obtain

$$l''(amtB(t_1)) = 0 .$$

Plugging 0 for both $l''(amtB(t_1))$ and $g''(amtB(t_1))$ in the equation for $g''(amt)$, we get

$$\frac{-1}{2}\big(l(amtB(t_1))\big)^{\frac{-3}{2}}.\big(l'(amtB(t_1))\big)^2 = 0 .$$

Since $l' > 0$,

$$\frac{-1}{2}\big(l(amtB(t_1))\big)^{\frac{-3}{2}} = 0,$$

which has no finite solution, indicating that there is no level corresponding to the amount in tank B at t_1, which is an absurdity. Since the only additional assumption we made in this line of reasoning was the existence of the derivatives of the l function, we have proven that, in this simulation, tank B can have no "smooth" shape, that is, one governed by a "level" function $l(amt)$ which is twice differentiable all through its domain. In this example, it is interesting to note that the sign-equality assumption, which is essentially a "smoothness" assumption about the underlying functions, rules out the possibility of another functional relationship in the system being smooth in a somewhat similar sense.

Conclusion

Our analysis has shown that the restrictions imposed on the simulated model by the sign-equality assumption (at least in the domain of liquid flow among containers) are very harsh: No shapes with uniform cross sections are covered for the tanks (except the topmost one) in the cascade examples. No "smooth" shape is covered for the second tank in the open-ended U-tube simulation. The systems that actually satisfy the additional restrictions (and thus get validly simulated) are an unrealistically small subset of the subject domain.

This leads us to conclude that the chattering and spurious behavior elimination problem for systems of connected tanks with reasonable shapes is still open, and one should treat the output of qualitative simulation with HOD constraints in *any* domain with additional care.

It should be noted that other aspects of simulation with HOD constraints (identification of chattering variables, filtering given an HOD sign, etc.) are sound and useful contributions to the qualitative reasoning repertory, and can be used fruitfully in conjunction with a method of determining the signs of the HODs without making the sign-equality assumption, for example, in a scheme where the user explicitly supplies the values or expressions to be used when applying the HOD constraint.

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