Qualitative reasoning on economic models

Christian Steinmann
Wirtschafts- und Sozialwissenschaftliche Fakultät, Universität Rostock
D–18051 Rostock, Germany
e-mail: christian@wiwi.uni–rostock.de

Abstract

Qualitative analysis has a long tradition in economic modeling. The application of mathematical methods, either quantitative or qualitative, to analyse an economic model stipulates strong assumptions. These assumptions restrict the analysis on a special case of the more general, verbally stated model. Furthermore the application of these methods is successful in analysing small models only. In this paper it is discussed, whether a qualitative formulation of an economic model and its qualitative analysis via qualitative reasoning will result in statements describing the behavior of the model. Doing this, economists can avoid the pitfalls of having to state the model as a concrete system of differential equations. However it is not expected to achieve different results. Beyond this, maybe more complex models can be analysed qualitatively.

Introduction

Whole seen the real economic process is extremely complex and difficult to survey. This paper is not the right place to analyse the complexity itself, to aid further discussion I only want to give a short glimpse into the problem, first of all. Each household and each member of a household decides how to act in the economy. These decisions are the foundation of the business recession and revival, of stability and growth of an economy. They are influenced by many economic variables. On the other hand the decisions themselves will change the value of these variables. The interaction between economic variables and decisions of economic subjects, in combination with the vast number of economic subjects and goods, prevents the description and analysis of the real process in detail. The elementary operations in economic acting are producing, selling, and buying of goods. In principle each kind of good has one market and one price. In addition, firms try to distinguish their products from those of their competitors. Hence, goods of the same kind differ in quality, service, color, or equipment. Therefore on every goodsmarket there are goods marked with differing features and dealt with different prices. There are hardly any homogeneous goods. Adjustment of prices and quantities leads to an equilibrium in production, supply, and demand of goods. It would be hard to reason why this adjustment should take place in the same way, and with the same speed on every market. The adjustment of prices and quantities in one market will be influenced by other external variables than those influencing the process in another market. The sum of all adjustments in every single market for goods or production factors forms the dynamics of the entire economy. Hence, it is impossible to give exact prediction of the real economic progress, since it is impossible to describe the real economy in a model which can be analysed. By the way, it is not enough to look at the economy of a single country, an economy has to be investigated worldwide, apart from the problem that the *real* functional relations between economic variables and decisions of economic agents are unknown.

The crucial question concerning this microbased view on the economic process is whether suitable assumptions can be found to describe the behavior of the economic agents. The economist has to deal with the same question like sociologists or phsychologists as TESCH (18) asks: *"Is the human/social world, just like the physical nature, governed by natural 'laws'? Or is human behavior something we can not predict, something onto whose inner workings we can only shed more or less light in our effort to understand."* On the other hand, also in economics the whole thing is more than the sum of its parts. The physicist can not predict the behavior of a single electron in interaction with others. Despite of the fact that every electron behaves according known laws, a real system is too complex to be analysed. But the behavior of the whole system, for example in thermodynamics, can be predicted. Obviously the economic process does not behave like a chaotic system. Without complete knowledge it can be assumed that the system behaves according certain laws although the behavior of every single economic agent is not completly known.

A central task of economics is to identify the basic laws of the underlying economic process. For this purpose the goal of economic research is not to represent reality completely. Instead only small cuts are considered. Doing this, unrealistic assumptions have to be accepted to restrict the model on such a small cut. However, the main parts of reality have to be selected, and other parts are excluded with the help of these assumptions. The result is a large gap between the real world and the model. GOODWIN states in his article "A growth cycle" in a similar context: *"Any ... economist should ask: why analyse an unreal, idealized system? The answer is that to show the logic und plausibility of a type of behavior and of its analysis, it is essential to get it clearly und simply stated."* Accordingly an unrealistic, idealistic model is inspected in order to determine certain aspects. In a more realistic model these aspects would otherwise be superimposed by other effects. Additionaly, it would be impossible to solve by analytical methods. This problem defeats most attempts to verify economic models by econometric methods empirically. Every model, and especially every economic model, has to deal with a big number of variables as exogeneous constants. An econometric analysis of such a model has to filter out exogeneous shocks which happens in reality because these variables are not constant.

The problem in this paper is the analysis of an economic model. The economist states assumptions to build a model like Solow's stable full employment model, described in the next section. His further task is to prove the dynamic behavior of the economic process defined by that model. Will this model lead for example to stable full employment, to stable under employment, or will the process be unstable?

Qualitative methods in mathematics

Economic modeling and model analysis are not imaginable without the application of qualitative methods. Proceeding from a verbal model formulation the resulting growth cycle will be described verbally, too. It is stated and argued in linguistic terms like *high*, *low*, *increasing*, or *decreasing*. On the other hand, the formal description of dynamic economic systems in form of differential equations is based on the work of Walras, see (13) for example. The formal notation in differential equations ensures an unequivocal model description and allows analysis with mathematical methods. However, these differential equation systems are investigated qualitatively. *"By qualitative analysis we mean the analysis of the general properties of the model, abstracted from the single numerical case"*, says GANDOLFO (6). According to Gandolfo qualitative Analysis divides in *i)* fixing the equilibrium point, *ii)* comparative static analysis, *iii)* equilibrium analysis and *iv)* analysis of parameter value sensitivity. Statements concerning the dynamic behavior of a model are derived without quantifying the parameters. To apply qualitative methods of mathematical analysis it is necessary to accept some restricting assumptions as stated above. Especially linear coherences are often assumed. When some distinct relations are expressed by linear differential equations, these formal notation is describing a limited special case of the more general model which was expressed and specified in verbal terms. The results of the qualitative analysis of the differential equation system are used to analyse and to explain the behavior of the verbal model. However, this paper concentrates on Gandolfos part *iii)* equilibrium analysis neglecting the others, mostly.

For example, the fundamental equation of Solow's stable full employment growth model is:

$$\hat{l} = n - sf(l) \qquad (1)$$

The growth rate of labor intensity $\hat{l}$ equals the growth rate of labor supply n less the growth rate of the capital stock $sf(l)$. The marginal propensity to save s is assumed constant in the range $[0,1]$. $f(l)$ denotes the neoclassical production function in its intensive form expressed by means of labor intensity. The original production function is noted as $F(K,L)$, production is a function of labor and capital input. The following assumptions conerning the production function are made:

$$f' > 0 \quad \text{and} \quad f'' < 0$$

or

$$F_K > 0, \quad F_L > 0,$$
$$F_{KK} < 0, \quad F_{LL} < 0 \quad \text{and} \quad F_{KL} = F_{LK} > 0.$$

I.e. the slope of the function is strictly positive and decreasing. This notation describes the shape of the function qualitatively. A concrete instance of the function, like $f(l) = 0.5\sqrt{l}$, is not given. The Solow model is analysable by a qualitative equilibrium analysis. After an exogeneous shock has happened, the process develops towards the (new) full employment equilibrium. Under the stated assumptions, the equilibrium of the model is globally asymptotically stable, see (4). To conclude this from the model, it is not necessary to impute a concrete shape of the production function. Such an assumption has only to be made for a numerical analysis which becomes necessary in a different, more complex model where no statements result when using the above mathematical methods.

On the other hand, strong assumptions are already stated to enable the application of mathematical analysis. At least the concrete-*form* of the differential equations has to be assumed anyway. Normally, linear or very simple coherences are stated. Savings in the Solow model are assumed as $S = sY$ with $s \in [0,1]$ as the constant marginal propensity to save. This is undoubtly a strong assumption, but it is presumed that the main topics of the model do not depend extensively on this assumption.

For another example consider the cross-dual adjustment rule for price and production in a simple market clearing model. The adjustment rule is denoted *cross-dual* because of the assumed crossing effects. Quantity affects the price development, price affects the quantity development:

$$\begin{aligned} \dot{p} &= \alpha(Y^d - Y^s) \\ \dot{Y^s} &= \beta(p - c) \end{aligned} \tag{2}$$

Again linear coherences are assumed. Excess demand $Y^d - Y^s$ affects the price with a constant adjustment speed α. An additional unit demand increases the price p, independently of the price level or the level of demand and supply. Accordingly, the excess of the price over the production costs per unit c affects the supply Y^s with a constant adjustment speed β. The analysis of the model by application of the Routh–Hurwitz theorem yields a stable equilibrium, if the demand is decreasing in changes of the price, or the production costs per unit are increasing in changes of supply, the production quantity. Concerning the Jacobi matrix

$$\mathcal{J} = \begin{pmatrix} \alpha Y_p^d & -\alpha \\ \beta & -\beta c_{Y^s} \end{pmatrix}$$

Y_p^d has to be negative, or c_{Y^s} has to be positive. In the case of a price independent demand, $Y_p^d = 0$, and fixed production costs, $c_{Y^s} = 0$, analysis, with the additional application of an appropriate Liapunov function, yields neutral oscillation, see for example (2) or (9).

Without quantifying the parameters α and β in the stated market clearing model, or s in the Solow model above, the chosen mathematical notation contains strong assumptions. In this way, the intended behavior of the economic system is fixed beyond the scope of a verbally stated model. Maybe it should be stated that the price is increasing with the excess demand, or that savings are increasing with income under the restriction that savings have to be smaller than income. Formal modeling has to solve the problem to find a model description, which fulfills the presuppositions for the application of qualitative mathematics, imputing only such assumptions which do not falsify the intended core of the general verbal model. It is easy to imagine that there are assumptions, possibly stated in 'noting a system of differential equations, which will yield effects that are characteristic for this special case only, and do not represent the general behavior of the model.

Adding these points concerning economics I have to go a bit further than for example SIMON does in his preface to the book "Qualitative Simulation Modelling and Analysis" (3): *"Sometimes, we know the shape of the equation that governs the phenomena of interest, but we do not know the numerical value of parameters — perhaps, at most, we know their signs."* I think at least in economic modeling not even the shape of the equations is known, virtually. The microeconomic behavior of economic agents can only be described verbally, resulting in qualitative effects on economic variables, or describing macroeconomic coherences between economic variables. Generally, only the direction of the effects is known. Further assumptions that fix the shape of the equations are stated for the sake of applicability of the mathematical methods. When those assumptions have been stated, then there are parameter values, not earlier.

Numerical simulation

In various fields of science and practice simulation is used to examine and visualize complex processes, or systems, through simplified models. It is beyond the scope of this paper to state the aim and purpose of modeling and simulation in general. I presume every well disposed reader of this paper has a sufficient, maybe intuitive, insight into the application of simulation methods.

Running a simulation on a computer, the algorithm deals with numerical data. These are moments of the distribution of duration or interarrival time, or other features of objects in discrete simulation, or values of level variables, parameters, and rates of adjustment in quasi continuous simulation. Of what kinds are the aims of simulation?

In several situations the objective of simulation is parameter optimization. Here the aim is a *numerical* result. There are many examples describing this kind of application. A typical one is the control of a road crossing by a traffic light system.

In other possible situations simulation is used as an aid for decision. Someone has to choose one of several alternatives and he is interested to choose the best or at least an appropriate alternative. In this case the numerical result is meaningless, and interesting only in comparison to other results. Maybe there is additionally a point of interest whether the numerical value hits a special range. A typical example here is the arrangement of machines to install a production process with some choosable alternative arrangements. So, the target of simulation is to find, according to some given criteria, the best possible alternative under a given system of restrictions. The numerical result of the simulation has to be classified, ordinally.

Finally there are other situations without any numerical input data available. To run a simulation those variables have to be quantified, artificially. Here a special case is modeled while a general case is the point of interest. Usually, numerical results are meaningless in such situations. Qualitative inputs are transformed into numerical data, and the numerical results have to be interpreted qualitatively. There are several applications in natural science, in physics for example, and in economics, especially. The application of qualitative simulation is straight forward.

Applying simulation, it is possible to analyse more complex models which can not be analysed directly by mathematical methods. An interdependent economic model containing more than three differential equations usually can not be analysed by qualitative methods of mathematical analysis. Maybe it is possible to describe the behavior of subsystems. Otherwise it is necessary to add additional assumptions for yielding at least restricted results. For numerical simulation of such a model again the model variables have to be represented in numerical values. A concrete situation is considered as a special case of the more general, formally noted model. See (14) for example, for numerical methods used with differential equations.

As stated above, the numerical result yielded by a simulation run is not the aim, but an intermediate step. The result is to be interpreted qualitatively. The main purpose in order to analyse the behavior of the more general model is to generalize the result. Generalizing numerical results always runs the risk of losing, if the results are specific only for this very limited special case. Inspecting a numerical model compared to a differential equation system is a stronger specialization than considering a differential equation system compared to a verbal model.

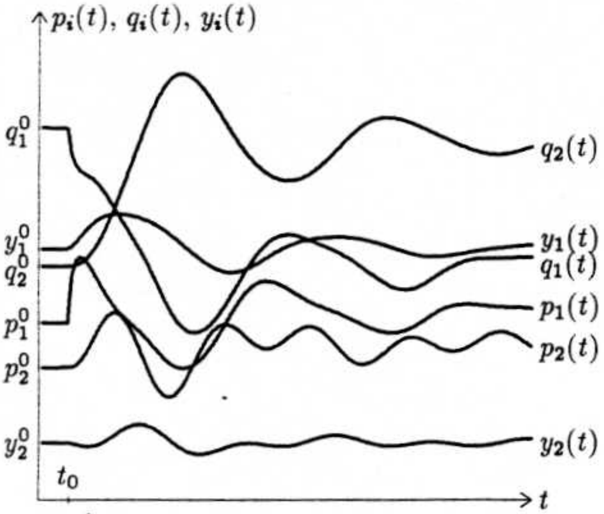

Figure 1: Numerical simulation

To figure a simple example I extend the differential equation system stated in equation 2. An economy containing two goods and two factors with cross–dual adjustment has to be analysed. It is stated that households consume good 1 and factor 1. These are useful for the households. The utility function is of Cobb–Douglas type. In this way, supply of factor 2 is constant. Good 2 and factor 2 are used together with good 1 and factor 1 as inputs to produce the two goods. The produced quantities are y_1 and y_2. The prices of goods p_1 and p_2 and of factors q_1 and q_2 develop in reaction on excess demand. The produced quantities are affected by the surplus of price against costs, see (13) for more details on cross–dual adjustment. Figure 1 represents the results of a numerical simulation of this model, in reaction on an exogeneous shock reducing supply of factor 2. The adjustment process takes a violent cyclic course to reach a new stable equilibrium point. Slight modifications of the model yield in significant changes in adjustment (17). If for example households consume all goods and factors, or if there are firms with a different production structure, the oscillation is damped. After an equal exogeneous shock the new equilibrium point will be reached much faster.

The qualitative approach

The main problem in analysing a verbal stated model is the limited possibility to receive unequivocal results by verbal argumentation. It is not a simple task to consider all effects between the economic variables completely. Additionally, a verbal model representation is usually ambiguous. For these reasons economists analyse a special case of the model depicted as differential equation system. Figure 2 shows the proceeding to analyse an economic model.

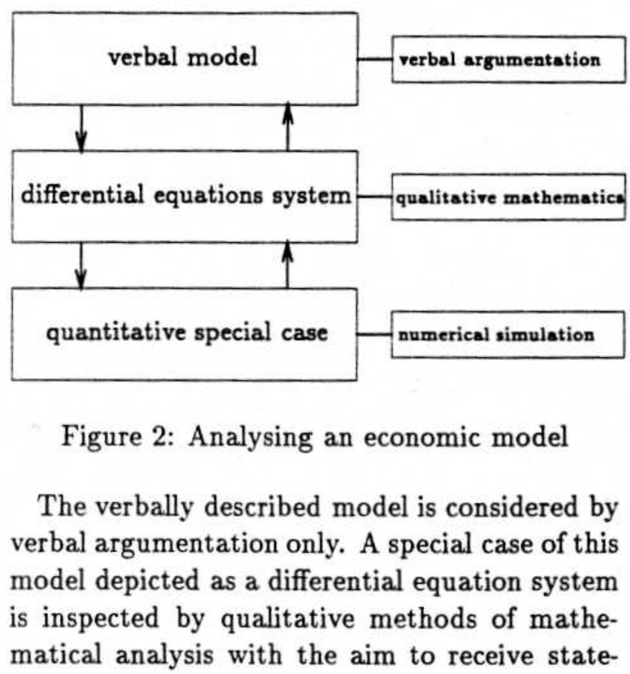

Figure 2: Analysing an economic model

The verbally described model is considered by verbal argumentation only. A special case of this model depicted as a differential equation system is inspected by qualitative methods of mathematical analysis with the aim to receive statements describing the behavior of the more general model. Finally a quantitative special case of the differential equation system is considered by numerical simulation, if mathematical methods miss applicability. The arrows directing downwards stand for the development of a special case, the other ones directing upwards express the attempt to generalize the results.

The qualitative approach, analogous to qualitative physics, see (8), or qualitative process theory, see (5), extends the analysis of economic models. A new level of analysis is included. Grantham and Ungar state in their article "Qualitative physics" considering physical processes: *"The aim of qualitative reasoning is to create and use representations of the world which are simplified so that irrelevant detail is ignored while maintaining enough resolution to distinguish and explain important features of behavior."*

This approach contains a formal notation without imputing the shape of functions or linear coherences. The methods of qualitative simulation and qualitative reasoning support this approach by making appropriate tools availiable for analysis. Without these methods a formal notation would be worthless at all. The corresponding proceeding of analysis can be seen in figure 3.

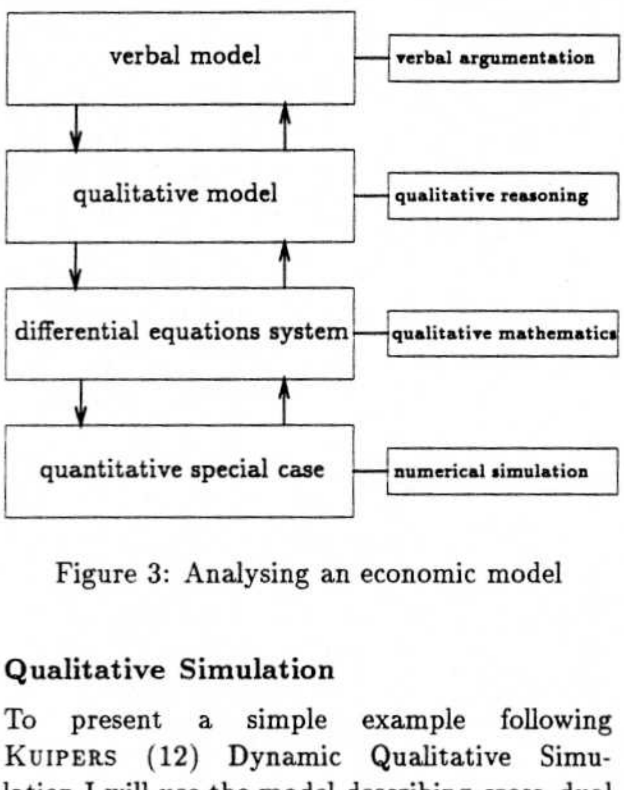

Figure 3: Analysing an economic model

Qualitative Simulation

To present a simple example following KUIPERS (12) Dynamic Qualitative Simulation I will use the model describing cross–dual adjustment of price and supply, which had been stated similarly, via differential equations in (2) on page 3. It has been analysed by mathematical methods and numerical simulation in the corresponding sections ahead. A negative effect of return per unit $r = p - c$ on production Y^s is assumed. c is the constant cost per unit output. Excess demand $Y^e = Y^d - Y^s$ has a positive effect on the price p and as a consequence on return, too. Furthermore demand Y^d is a negative function of price p. Qualitative functions are denoted $\mathcal{F}$ in this paper.

$$Y^s \leftarrow r \qquad 0\ldots\infty \qquad (3)$$

$$p \leftarrow Y^e \qquad 0\ldots c\ldots\infty \qquad (4)$$

$$Y^e = Y^d - Y^s \qquad -\infty\ldots 0\ldots\infty \qquad (5)$$

$$Y^d = \mathcal{F}(-p) \qquad 0\ldots\infty \qquad (6)$$

$$r = p - c \qquad -\infty\ldots 0\ldots\infty \qquad (7)$$

Values of return and excess demand may be positive, negative or zero. Price may be in the range $(0, c)$, equal to c or larger than c. Zero and c are special points in the value range of these variables. They mark turning points in the adjustment process. Like c zero is a threshold, called *landmark value* in the qualitative context. Values can be increasing, decreasing or constant. The states of the variables return and excess demand are characterized by two values: First the actual range or landmark value respectively, and second the direction of movement (increasing $\uparrow$, decreasing $\downarrow$, constant $-$, or indeterminate ?).

Simulation begins in some start situation, for example in a situation characterized by positive return and negative excess demand. Equations (3)–(7) give the corresponding values of the other variables and their directions of movement. Price p starts in the range (c, ∞). The other variables, Y^s, and Y^d, are bound to a single qualitative value, the range $(0, \infty)$, during simulation. Y^s and p are dynamically depended variables. Their direction of movement is given by the value of the variables r and Y^e on the right side of equation (3) and (4): Price will increase with excess demand, supply will increase in reaction on positive return. Direction of movement of the other variables in (5)–(7) is given by the direction of movement rather than value of the right side of these equations: Demand is increasing and return is decreasing with decreasing price. Direction of excess demand is indeterminate while supply and demand are both increasing.

The complete situation can be described by the following tuples:

$$Y^s = \{(0, \infty); \uparrow\} \qquad r = \{(-\infty, 0); \downarrow\}$$

$$p = \{(c, \infty); \downarrow\} \qquad Y^e = \{(-\infty, 0); ?\}$$

$$Y^d = \{(0, \infty); \uparrow\}$$

Proceeding from that state, price will reach landmark value c. No other move is possible in this simple model. With price equal to costs, supply will remain constant. Price will move on decreasing, forcing decreasing return and increasing demand. Excess demand will increase with increasing demand and constant supply. I will skip the further process of qualitative simulation in this paper, because the method is well known.

The dynamic course is shown in figure 4. The diagram a) in figure 4 shows the course of Y^e and r. This diagram is not appropriate, obviously. Excess demand is still moving, even if return equals zero. Diagram b) shows the interaction of the dynamic variables Y^s and p. The isoclines are figured for a better understanding.

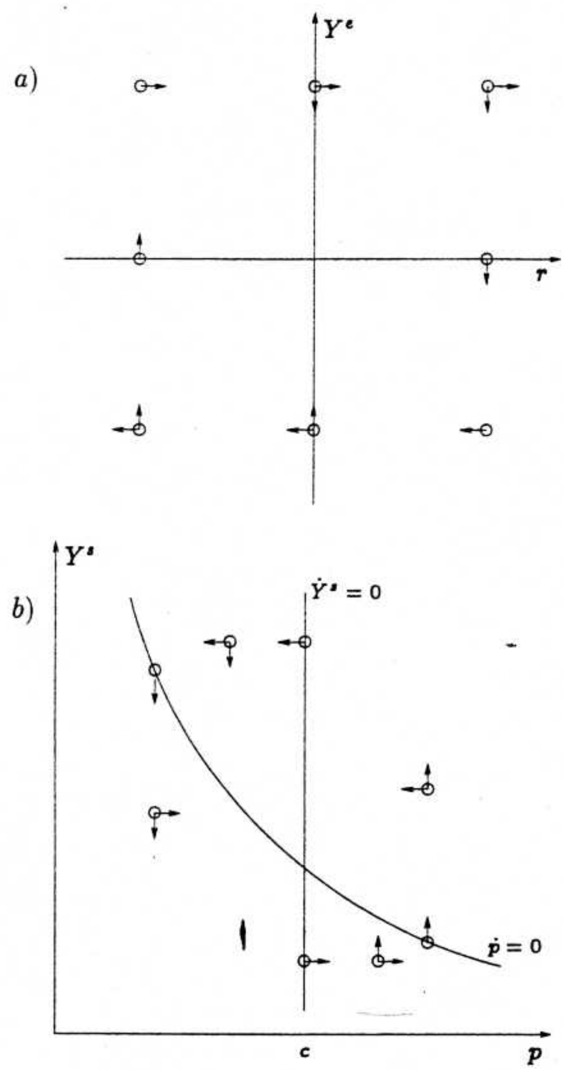

Figure 4: Qualitative simulation

The figures just mark one point: This qualitative simulation is not able to show whether the dynamic process will reach a stable equilibrium, will explode or keep a constant orbit. The cyclic characteristic of the dynamic behavior on the other hand is obvious. With the assumed negative effect of price on demand the adjustment process should result in a stable equilibrium, see the mathematical analysis above. The cycles only remain on a constant orbit, if the $\dot{p} = 0$ isocline lays parallel to the p–axis, if demand is independent of price. The qualitative values are too coarse to express damped cycles.

Qualitative simulation and orders of magnitude

To achive more specific results qualitative values must be more expressive to analyse a process aproaching cyclic an equilibrium. The assumptions in the stated example are not sufficient to compare the crucial values, especially the values of demand and supply. Both remained in the meaningless range $(0, \infty)$ during simulation. So, if demand is not equal to supply, price moves, independent of the amount of excess demand. In the egg market for example, price would rise in the same way in reaction to an excess demand of ten eggs or of ten million eggs. It would be much more plausible to assume, price will change very slowly, if at all, when excess demand is near zero. Then demand and supply are of the same *order of magnitude*.

Following RAIMAN a scale *Close* is introduced, including scale *Id*, see (16) for the details. Until now, a market has been called cleared if the single element sets *Id*(*variable*) overlaped:

$$Id(Y^d) \approx Id(Y^s) \quad \Leftrightarrow \quad Y^d = Y^s$$

To analyse an economic model a market now is assumed to be cleared, if the sets *Close*(*variable*) overlap:

$$\begin{aligned} &Close(Y^d) \approx Close(Y^s) \quad \Leftrightarrow \\ &\{y \mid (1-\varepsilon)Y^d < y < (1+\varepsilon)Y^d, \varepsilon > 0, \varepsilon \text{ small}\} \cap \\ &\{y \mid (1-\varepsilon)Y^s < y < (1+\varepsilon)Y^s, \varepsilon > 0, \varepsilon \text{ small}\} \\ &\neq \emptyset \end{aligned}$$

To compare these coarse values the $\approx$ relation was introduced for equal orders of magnitude.

As usual $=$ is true for equal values. The relation $<$ denotes smaller values in order of magnitude.

Applied on qualitative simulation this yields in different adjustment of dynamic variables if the determining value on the right hand side of the respective equation is *Close*(0). The value of Y^e is in *Close*(0) if $Close(Y^d) \approx Close(Y^s)$ holds.

To continue the example above the qualitative description of the model remains unchanged to the set of equations (3)–(7). In addition those pairs of variables are grouped together which has to be compared in orders of magnitude during the qualitative simulation process:

$$Y^d \overset{?}{\approx} Y^s \tag{8}$$

If c is not assumed to be constant but a function of production quantity, $p \overset{?}{\approx} c$ would have to be added to 8. However, it would not change the following simulation if $p \overset{?}{\approx} c$ is added despite c constant.

The start situation is the same as above, added the order of magnitude relation between Y^d and Y^s:

$$\begin{aligned} Y^s &= \{(0,\infty);\uparrow\} & r &= \{(-\infty,0);\downarrow\} \\ p &= \{(c,\infty);\downarrow\} & Y^e &= \{(-\infty,0);?\} \\ Y^d &= \{(0,\infty);\uparrow\} & Close(Y^d) &< Close(Y^s) \end{aligned}$$

The first simulation step yields price equal to costs. Supply keeps constant, while demand increases. The negative excess demand moves towards zero:

$$\begin{aligned} Y^s &= \{(0,\infty);-\} & r &= \{0;\downarrow\} \\ p &= \{c;\downarrow\} & Y^e &= \{(-\infty,0);\uparrow\} \\ Y^d &= \{(0,\infty);\uparrow\} & Close(Y^d) &< Close(Y^s) \end{aligned}$$

Decreasing p leaves landmark value c. The distance between demand and supply is reduced because of increasing demand. Prices adjustment slows down.

$$\begin{aligned} Y^s &= \{(0,\infty);\downarrow\} & r &= \{(-\infty,0);\downarrow\} \\ p &= \{(0,c);\downarrow\} & Y^e &= \{(-\infty,0);\uparrow\} \\ Y^d &= \{(0,\infty);\uparrow\} & Close(Y^d) &\approx Close(Y^s) \end{aligned}$$

Excess demand is equal to landmark value zero. Price keeps constant below costs, demand keeps konstant, too. Supply decreases because of negative return:

$$\begin{aligned} Y^s &= \{(0,\infty);\downarrow\} & r &= \{(-\infty,0);-\} \\ p &= \{(0,c);-\} & Y^e &= \{0;\uparrow\} \\ Y^d &= \{(0,\infty);-\} & Id(Y^d) &\approx Id(Y^s) \end{aligned}$$

Excess demand leaves landmark zero, demand is still near supply. Price starts to increase. Supply and demand decreases:

$$\begin{aligned} Y^s &= \{(0,\infty);\downarrow\} & r &= \{(-\infty,0);\uparrow\} \\ p &= \{(0,c);\uparrow\} & Y^e &= \{(0,\infty);?\} \\ Y^d &= \{(0,\infty);\downarrow\} & Close(Y^d) &\approx Close(Y^s) \end{aligned}$$

In this crucial situation the damping effect of price elastic demand does its work. With decreasing supply and demand, excess demand keeps positive but close to zero. Price adjusts slowly increasing until it is equal to costs.

$$\begin{aligned} Y^s &= \{(0,\infty);-\} & r &= \{0;\uparrow\} \\ p &= \{c;\uparrow\} & Y^e &= \{(0,\infty);\downarrow\} \\ Y^d &= \{(0,\infty);\downarrow\} & Close(Y^d) &\approx Close(Y^s) \end{aligned}$$

Simulation does not stop in this situation. Still some variables will move. But the further steps, which I have skipped here, result in a cycle close to the equilibrium. The process will not leave the range close to the equilibrium again. Qualitatively, the adjustment process stops in equilibrium.

Figure 5 shows the dynamic course. The thick lines are the respective isoclines. The range between the thin lines to the right and left of the thick lines is *close* to the isoclines. The intersection of both ranges is close to the equilibrium. For a better understanding the range close to the $\dot{Y}^s$=0 isocline is added, despite the fact that price and costs have not been compared in orders of magnitude during simulation.

Simulation starts in the point right to both isoclines. Unlike above it is obvious that the cycles are damped. After leaving the $\dot{p}$=0 isocline adjustment does not leave the range close to that isocline again. That is because of the stabelizing effect of price elastic demand.

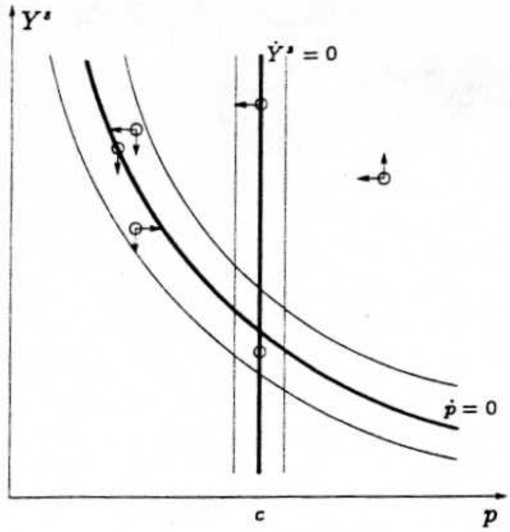

Figure 5: Qualitative simulation and orders of magnitude

Conclusion

The application of qualitative reasoning on economic models appears downright forcing in comparison to the application on physical processes. In economics the basis of proved physical laws is missing. In contrast to natural science, in economics it is not possible to investigate or prove basic laws under laboratory conditions. Missing or incomplete data is not the main characteristic of economic analysis. Real data are no object in the context of equilibrium analysis. Relationships are assumed, and according to these assumptions the economist analyses the behavior of the corresponding system.

From this point of view the critique of CELLIER in (1) on the qualitative physics approach has to be weakened, but remains a motivation for future work. Cellier compares the qualitative simulation of a differential equations system according to KUIPERS (11) with an analysis of the same system by qualitative mathematics. It is not remarkable that the results of qualitative simulation are less specific. Especially qualitative simulation is not able to state parameter relations depicting the relations yielding in stable or unstable behavior of the model.

In his example Cellier describes a system of three linear differential equations with parameters a, b, and c. The mathematical analysis of the model yields a stable adjustment if $c < ab$ holds. In comparison with that the system is transformed into qualitative notation. Kuipers QSIM algorithm grants to find every type of behavior that is physically possible. In this example it will be the result that stable, unstable, or neutral behavior is possible. The qualitative description of the model does not allow statements beyond these. Cellier says, *"Unfortunately, this does not tell us very much."* The fact that this chosen model formulation allows every noted kind of adjustment, is already a result. On the other hand this result is not sufficient. The next step is to find the conditions which determine the kind of behavior. In this paper it is done via adding orders of magnitude to qualitative simulation.

Future Work

In contrast to the stated example above, in more complex models more than only one move will be possible in a certain qualitative situation. The simulation of a qualitative model is directly analogeous to the application of a production system. This can be used for following different paths. Production systems implement knowledge based systems. The major components are *i)* a collection of rules, each of which is composed of a condition and an action, *ii)* a working memory which contains the developing information that defines the current state of the system, and *iii)* a control loop which cycles continually. In every cycle one rule is applied. The conditions of the rules are matched against the information in the working memory. A rule whose conditions evaluate as true can *fire*, see (10) or (15) for example. In our case the rules are the qualitatively noted coherences between the economic variables in the working memory. In every situation defined by given values of the economic variables, the presuppositions for reaction of more than one variable hold. The resulting behavior of the system depends on the decision, as to which variable will react first. Because of the combinatorical explosion of the number of paths that would have to be considered, it is impossible to inspect every possible development. However, the inspection of a path can be stopped when a special behavior is discovered. It is to assume that such a behavior will be recognizable after a few production cycles.

The chosen conflict resolution method decides which variable will react faster than the others, if several variables can react in a situation. Inspecting and modifiying the corresponding parameters of the conflict resolution mechanism can help to identify the conditions of model behavior. A statement like "if variable x reacts faster than variable y this will result in explosive behavior" can be expected. It has to be studied whether more complex coherences can be achieved, like e.g. the parameter based statement $c < ab$.

References

(1) François E. Cellier. General system problem solving paradigm for qualitative modeling. In Paul A. Fishwick and Paul A. Luker, editors, *Qualitative Simulation Modelling and Analysis.* Springer Verlag, New York, 1991.

(2) Lev E. El'sgol'c. *Qualitative Methods in Mathematical Analysis.* American Mathematical Society, 1964.

(3) Paul A. Fishwick and Paul A. Luker. *Qualitative Simulation Modelling and Analysis.* Springer Verlag, New York, 1991.

(4) Peter Flaschel. *Macrodynamics: income distribution, effective demand and cyclical growth.* Verlag Peter Lang, Frankfurt/M., 1993.

(5) Kenneth D. Forbus. Qualitative process theory. In Daniel G. Bobrow, editor, *Qualitative reasoning about physical systems.* North Holland, 1984.

(6) Ciancarlo Gandolfo. *Qualitative Analysis and Econometric Estimation of Continuous Time Dynamic Models.* North–Holland, Amsterdam, 1981.

(7) Richard M. Goodwin. A growth cycle. In J.G. Schwarz E.K. Hunt, editor, *A Critique of Economic Theory.* Penguin Books, Baltimore, 1972.

(8) Stephen D. Grantham and Lyle H. Ungar. Qualitative physics. In R.B. Banerji, editor, *Formal Techniques in Artificial Intelligence.* North Holland, 1990.

(9) Morris W. Hirsch and Stephen Smale. *Differential Equations, Dynamical Systems, and Linear Algebra.* Academic Press, New York, 1974.

(10) P. Klahr and D. Waterman. *Expert Systems.* Addison-Wesley, New York, 1986.

(11) Benjamin J. Kuipers. Qualitative simulation. *Artificial Intelligence*, 29:289–338, 1986.

(12) Benjamin J. Kuipers. *Qualitative reasoning: modeling and simulation with incomplete knowledge.* MIT Press, Cambridge Mass., 1994.

(13) Michio Morishima. *Walras Economics.* Cambridge University Press, Cambridge Mass., 1977.

(14) James M. Ortega and William G. Poole. *An Introduction to Numerical Methods for Differential Equations.* Pitman Publishing Inc., Marshfield Mass., 1981.

(15) Derek Partridge and K.M. Hussain. *Artificial Intelligence and Business Management.* Ablex Publishing Corporation, Norwood, New Jersey, 1992.

(16) Oliver Raiman. Order of magnitude reasoning. *Artificial Intelligence*, 51:11–38, 1991.

(17) Christian Steinmann. *Analyse von Preis–Mengen–Anpassungsprozessen aus theoretischer und empirischer Sicht.* Diplom–Arbeit, Universität Bielefeld, 1992.

(18) Renata Tesch. *Qualitative research: analysis types and software tools.* The Palmer Press, New York, 1990.